Refined Motion Compensation for Highly Squinted Spotlight Synthetic Aperture Radar

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Abstract

This paper presents a motion compensation algorithm which is capable of dealing with large motion errors for highly squinted spotlight SAR. The approach consists of a phase error compensation, a shift to correct the delay time error and an interpolation/resampling in azimuth. For pulsed SAR with a 30° squint angle, the proposed algorithm compensates motion errors that are 10 times greater than the resolution cells and obtains nearly the theoretical achievable resolutions of the focused image. The main contribution of this paper is the consideration of the effect of motion errors in the presence of a large squint angle.

1 Introduction

Advanced synthetic aperture radar (SAR) systems are continually developing in the direction of more compactness in hardware size and higher spatial resolution. This makes their operation feasible on smaller airplanes or unmanned aerial vehicles. High spatial resolution requires methods for precise motion compensation (MoCom). If motion errors are not corrected, the image quality considerably degrades in terms of geometric resolution, radiometric accuracy and image contrast [1].

The motion error can be separated into two parts: an off-track error and a deviation in azimuth due to a variable velocity. Most MoCom approaches accomplish a so called first-order MoCom directly after range compression. The first-order MoCom corrects an incorrect velocity in azimuth by an interpolation and resampling [1] and corrects the off-track error in the mean squint angle direction [2]. Thereby, an additional deviation in azimuth originating from the squint angle remains uncompensated. In consequence for many SAR modes, e.q. highly squinted spotlight SAR, the first-order MoCom is not sufficient to fully compensate the motion error. Several works have been dedicated to the residual errors, most of them try to correct them via complex second or third-order MoCom within the SAR focusing algorithm [2, 3, 4].

This paper presents a refined first-order MoCom which is able to deal with large motion errors for highly squinted spotlight SAR. It is shown that by properly considering the effect of the squint angle on the off-track error, the motion error can be almost fully compensated within the first-order MoCom. The key idea is to correct the phase error independently from the time shift induced by an off-track error. It is shown that in case of a squint angle, the time shift correction of an off-track error involves a shift in the direction of view and a shift in azimuth.

In the next section, the SAR focusing technique with Omega-K algorithm is described. Afterwards, section 3 analyzes the motion error in squinted SAR geometry and presents the derivation of the proposed MoCom approach. Results and discussions are presented in section 4.

2 Squinted spotlight SAR focusing with Omega-K algorithm

The SAR data acquisition geometry is shown in Fig. 1. While the platform moves with a constant velocity v along the x-axis, the radar illuminates the ground within $x \in [-L/2, L/2]$. All scatterers can be considered to lie on a conceptual x-z plane, where the z-axis is orthogonal to the x-axis [5]. The x-axis is called azimuth and the z-axis slant range. In squinted SAR mode, the direction of view is rotated by the squint angle θ_s from the slant range direction z. The y-axis is called ground range and the h-axis denotes the height. The depression angle θ_D is defined in the y-h plane.

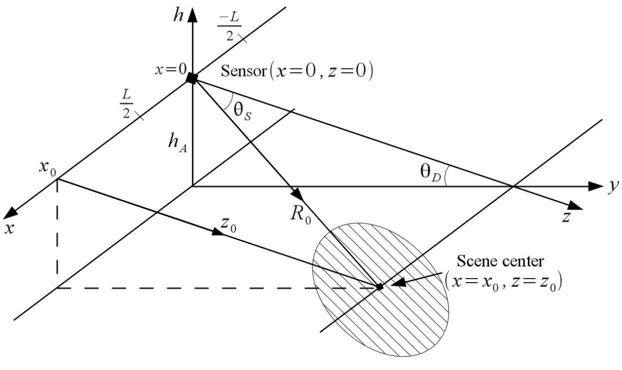


Figure 1: SAR geometry in squinted spotlight mode

At a slow time sample t_s the high frequency transmitted chirp signal $s_{HF}(t)$ is

$$s_{HF}(t) = e^{j\pi\gamma t^2} \text{rect}\left\{\frac{t}{T}\right\} e^{j2\pi f_c t}, \quad (1)$$

where γ is the positive chirp rate, T the pulse duration and f_c the carrier frequency. If the return from a point target p is received delayed by a time t_p, the received signal can be expressed as

$$s_{r,HF}(t) = e^{j\pi\gamma(t-t_p)^2} \text{rect}\left\{\frac{t-t_p}{T}\right\} e^{j2\pi f_c(t-t_p)}. \quad (2)$$

After quadrature demodulation via multiplication with $e^{-j2\pi f_c t}$, the baseband received signal is

$$s_r(t) = e^{j\pi\gamma(t-t_p)^2} \text{rect}\left\{\frac{t-t_p}{T}\right\} e^{-j2\pi f_c t_p}. \quad (3)$$

Range compression is achieved with a matched filter $h(t) = e^{-j\pi\gamma t^2} \text{rect}\left\{\frac{t}{T}\right\}$, resulting in the range compressed signal

$$s_p(t) = T\text{si}\{\pi\gamma T(t-t_p)\} e^{-j2\pi f_c t_p}. \quad (4)$$

The delay time t_p is a function of the slow time t_s, since the delay time $t_p(t_s)$ between the sensor and the point target changes with t_s while the platform moves within $x \in [-L/2, L/2]$. Thus, $e^{-j2\pi f_c t_p(t_s)}$ describes the azimuthal phase of the SAR data.
After range compression the Doppler frequency shift evoked by the squint angle θ_s is corrected. The range compressed, Doppler shift corrected data is then fed into the Omega-K algorithm. An Omega-K algorithm capable to focus highly squinted spotlight SAR data is presented in [6].
In case of existing motion errors, MoCom must be performed before the Omega-K algorithm, see Fig. 2.

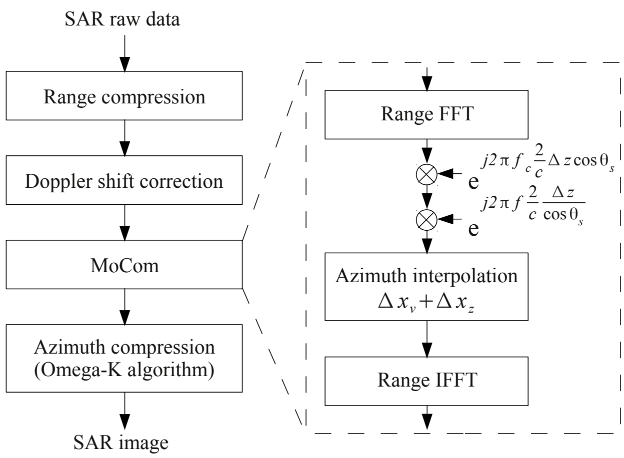


Figure 2: SAR focusing with the proposed MoCom and Omega-K algorithm.

3 MoCom for highly squinted SAR

MoCom can only be performed with accurate knowledge of the carrier flight track measured by the inertial measurement unit (IMU) and global positioning system (GPS). At every slow time sample t_s, the position error is calculated as the deviation of the actual carrier position from the nominal position on the ideal flight track. The position error is usually decomposed in three components in Cartesian coordinates $\Delta x_v, \Delta y$ and Δh. The deviation in slant range Δz, so called the off-track error, is calculated from the projections of Δy and Δh onto the z-axis: $\Delta z = \Delta y \cdot \cos\theta_D + \Delta h \cdot \sin\theta_D$, see Fig. 3. The azimuthal deviation Δx_v arises from a variable flight velocity of the carrier.
MoCom can be accomplished in two steps: first the compensation of Δz and second the compensation of Δx_v [1]. However, it has to be noted that MoCom can only performed in the direction of view (i.e. the direction of wave propagation) and in azimuth direction. In squint mode, the slant range direction differs from the direction of view by the squint angle θ_s.

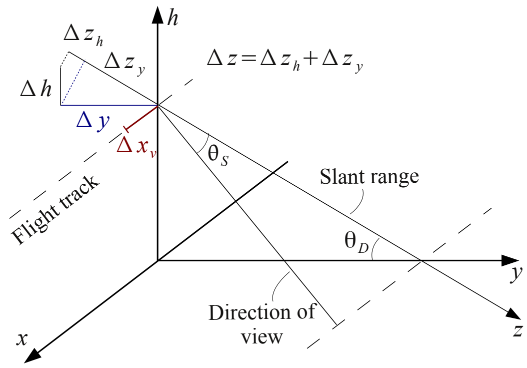


Figure 3: Position error measured with IMU/GPS

3.1 Correction of Δz

Fig. 4 shows the situation of an off-track error Δz of the sensor in slant range: The echo of the point target p is received at the incorrect position C instead of the correct position A. Δz results in a delay time error t_e of the received echo. However, t_e is linked to the distance deviation ΔR between target p and sensor position C via

$$t_e = \frac{2}{c}\Delta R = \frac{2}{c}\Delta z \cos\theta_s. \quad (5)$$

The error-prone, range compressed signal is

$$\tilde{s}_p(t) = T\text{si}\{\pi\gamma T(t-t_p-t_e)\} e^{-j2\pi f_c(t_p+t_e)}. \quad (6)$$

Comparison between (4) and (6) shows that the delay time error t_e induces a phase error $e^{-j2\pi f_c t_e}$ and a shift of the time function si$(\cdot)$ by t_e. The phase error can be compensated by a multiplication with

$$e^{j2\pi f_c t_e} = e^{j2\pi f_c \frac{2}{c}\Delta z \cos\theta_s} \quad (7)$$

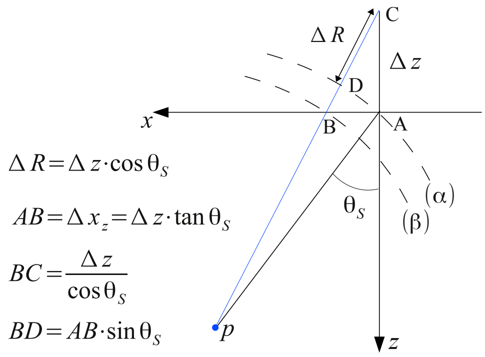


Figure 4: Off-track deviation Δz in the x-z plane

with t_e in (5), yielding

$$\tilde{s}_{p1}(t) = T\mathrm{si}\{\pi\gamma T(t - t_p - t_e)\}e^{-j2\pi f_c t_p}. \tag{8}$$

The time shift t_e of the si($\cdot$) time function cang be corrected by a multiplication with the corresponding phase in frequency domain:

$$\tilde{s}_{p1}(t) = T\mathrm{si}\{\pi\gamma T t\}e^{-j2\pi f_c t_p} * \delta(t - t_p) * \delta(t - t_e) \tag{9}$$

$$\begin{aligned}\tilde{S}_{p1}(f) &= \mathcal{F}\{\tilde{s}_p(t)\}\\ &= \frac{1}{\gamma}\mathrm{rect}\left\{\frac{f}{\gamma T}\right\} e^{-j2\pi f_c t_p} \cdot e^{-j2\pi f t_p} e^{-j2\pi f t_e}.\end{aligned} \tag{10}$$

At this point, it has to be recalled that the fast time domain t is linked to the direction of wave propagation (or the direction of view). Thus, only a shift in the direction of view can be implemented. In order to shift the incorrect position from point C to the correct position at point A, see Fig. 4, we first have to shift the incorrect position from C to B by a multiplication of $\tilde{S}_{p1}(f)$ with $H_{shift}(f)$:

$$H_{shift}(f) = e^{j2\pi f \frac{2}{c}\frac{\Delta z}{\cos\theta_s}} \tag{11}$$

$$\begin{aligned}\Rightarrow \tilde{S}_{p2}(f) &= \frac{1}{\gamma}\mathrm{rect}\left\{\frac{f}{\gamma T}\right\} e^{-j2\pi f_c t_p} e^{-j2\pi f t_p}\\ &\quad \cdot e^{-j2\pi f t_e} e^{j2\pi f \frac{2}{c}\frac{\Delta z}{\cos\theta_s}}\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{S}_{p2}(f) &= \frac{1}{\gamma}\mathrm{rect}\left\{\frac{f}{\gamma T}\right\} e^{-j2\pi f_c t_p} e^{-j2\pi f t_p}\\ &\quad \cdot e^{j2\pi f \frac{2}{c}\Delta z \tan\theta_s \sin\theta_s},\end{aligned} \tag{13}$$

whereby $e^{-j2\pi f t_e} = e^{-j2\pi f \frac{2}{c}\Delta z \cos\theta_s}$ and the relationship $\frac{1}{\cos\theta_s} - \cos\theta_s = \tan\theta_s \sin\theta_s$ is used.
Subsequently, the shift from B to A corresponding to $\Delta x_z = AB = \Delta z \tan\theta_s$ has to be realized, see Fig. 4. The correction of $-\Delta x_z$ is carried out by an interpolation and resampling in the azimuth direction. The resampling by $-\Delta x_z$ from B to A shifts the recording position from the wavefront (β) to the wavefront (α). This azimuthal shift automatically evokes a shift of $\Delta x_z \sin\theta_s$ in the direction of view, which can be expressed by a multiplication with $e^{-j2\pi f \frac{2}{c}\Delta x_z \sin\theta_s} = e^{-j2\pi f \frac{2}{c}\Delta z \tan\theta_s \sin\theta_s}$ in the frequency domain. Thus, the interpolation and resampling in azimuth automatically compensates the residual phase error $e^{j2\pi f \frac{2}{c}\Delta z \tan\theta_s \sin\theta_s}$ in (13) and yields the corrected signal as if it were recorded at the nominal position A

$$S_{p2}(f) = \frac{1}{\gamma}\mathrm{rect}\left\{\frac{f}{\gamma T}\right\} e^{-j2\pi f_c t_p} e^{-j2\pi f t_p} \tag{14}$$

$$\Rightarrow s_{p2}(t) = T\mathrm{si}\{\pi\gamma T(t - t_p)\}e^{-j2\pi f_c t_p}. \tag{15}$$

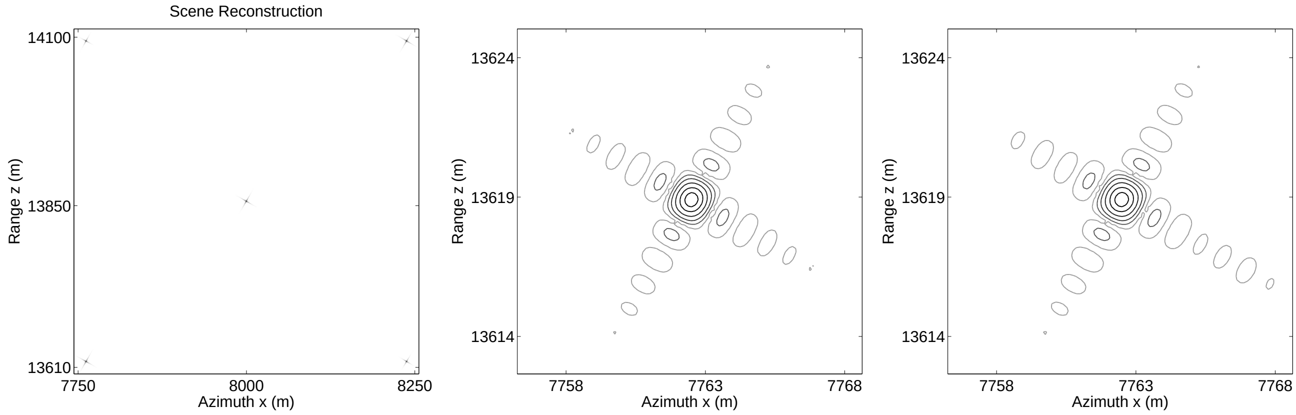
The result in (15) is identical with the error-free range compressed signal in (4).

3.2 Correction of Δx_v

In order to retrieve the spatial equidistant sampling in azimuth a resampling step is also required. This resampling step is realized by a interpolation in azimuth. The resampling procedure automatically compensates the phase error induced by the spatial deviation Δx_v in azimuth [1].
In practice, the corrections of Δx_v and Δx_z are carried out together in one interpolation and resampling step. The whole MoCom approach is shown in Fig. 2. The phase compensation in (7) and the correction of the time shift t_e can also be carried out during the range compression, which reduces the number of additional FFT's.
It has be emphasized that the order of all steps in the presented method has to be strictly kept. First, the Doppler frequency shift correction must be carried out in order to restore the orthogonal geometry of the SAR image. Secondly, the compensation of the delay time error t_e has to be done on the basis of every received pulse. In the last step, the variable sample spacing in azimuth can be corrected by an interpolation. After this step, the data is spatially equidistant but not temporally equidistant. [1]

4 Results

Fig. 5 presents the results of airborne X-band SAR simulations with parameters shown in Table 1. Five targets are distributed in a square region of the size 500m × 500m. With a reference range $R_0 = 16$ km and a squint angle $\theta_s = 30°$, the scene center is located at $(x_0 = 8\text{km}, z_0 = 13.856\text{km})$. SAR raw data from the five targets were simulated with and without motion errors.
In order to analyze the performance of the presented MoCom technique, significant motion errors in azimuth and range direction were included in the simulation. The deviation in range and azimuth from the ideal flight track was set to vary between ±10 m, see Fig. 6.
The simulated SAR raw data is submitted to the processing chain shown in the flow chart in Fig. 2. For raw data corrupted with motion errors, the presented MoCom is applied. Comparing the results of the image reconstruction from error-free data (in Fig. 5 (b)) and from the with MoCom corrected data (in Fig. 5 (c)), we can see that the motion errors are mostly compensated and the theoretical achievable resolutions in range and azimuth are almost attained.



(a) Square region with 5 targets ($\times$)

(b) Error-free data

(c) Results with presented MoCom for data corrupted with motion errors

Figure 5: Simulation results. In (b) and (c) the bottom left point target is zoomed.

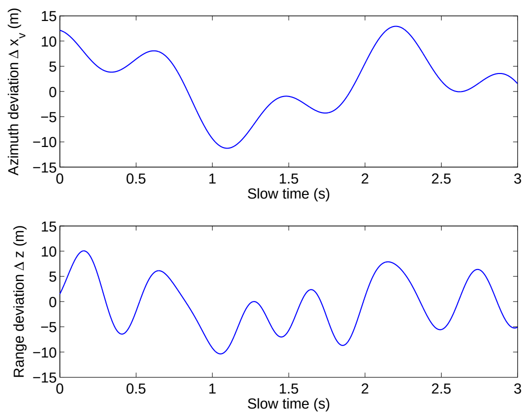


Figure 6: Simulated motion errors in azimuth and in slant range.

Table 1: Simulation parameters

Carrier frequency f_c	10 GHz
Pulse bandwidth B_p	150 MHz
Pulse duration T	6 μs
Sample frequency	180 MHz
Platform velocity v	100 m/s
Synthetic aperture length	300 m
Pulse repetition frequency	400 Hz

5 Conclusions

This paper presents a refined MoCom technique, which is capable to compensate large motion errors in focusing highly squinted spotlight SAR. The paper shows that besides the phase error correction, a shift is needed to compensate the delay time error. Additionally, in case of a squint angle, a correction in azimuth by an interpolation and resampling is as well indispensable.

The proposed MoCom is flexibly applicable since it is placed between range compression and the azimuth compression and therefore it is independent from the focusing algorithm of choice. Moreover, the algorithm is a generalization for variable squint angle and is also valid for broadside mode. So far this approach has been tested on X-band SAR data focused with Omega-K algorithm and needs further investigations on different settings to find its accuracy limitations.

References

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