Computing the Intrinsic Camera Parameters Using Pascal's Theorem

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Abstract. The authors of this paper adopted the characteristics of the image of the absolute conic in terms of Pascal's theorem to propose a new camera calibration method. Employing this theorem in the geometric algebra framework enables the authors to compute a projective invariant using the conics of only two images which expressed using brackets helps us to set enough equations to solve the calibration problem.

1 Introduction

The computation of the intrinsic camera parameters is one of the most important issues in visual guided robotics. One important method of selfcalibration is based on the image of the absolute conic and it requires as input only information about the point correspondences [5, 4, 1].

In this paper we re establish the idea of the absolute conic in the context of Pascal's theorem and we get different equations than the Kruppa equations [4, 1]. Although the equations are different they rely on the same principle of the invariance of the mapped absolute conic. The consequence is that we can generate equations so that we require only a couple of images whereas the Kruppa equation method requires at least three images [4]. As a prior knowledge the method requires the translational motion direction of the camera and the rotation about at least one fixed axis through a known angle in addition to the point correspondences. The paper will show that although the algorithm requires the extrinsic camera parameters in advance it has the following clear advantages: It is derived from geometric observations, it does not stick in local minima in the computation of the intrinsic parameters and it does not require any initialization at all. We hope that this proposed method derived from geometric thoughts gives a new point of view to the problem of camera calibration.

In this paper we are modelling the properties of the projective space $\mathcal{P}^3$ using the geometric algebra $\mathcal{G}_{1,3,0}$ and that of the projective plane $\mathcal{P}^2$ using $\mathcal{G}_{3,0,0}$. Next, we will briefly outline the basic operations of projective geometry within the geometric algebra which are used in the following sections. For a complete introduction of geometric algebra and algebra of incidence in computer vision the reader should consult [3].

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In an n-dimensional vector space with an orthonormal basis $\{\mathbf{e}_1, \ldots, \mathbf{e}_n\}$ we define an 2^n dimensional space consisting of $\mathbf{e}_1^{m_1} \ldots \mathbf{e}_n^{m_n} (m_j = 0, 1)$, where $\mathbf{e}_i^0$ is interpreted as the identity, and define a product on these basis vectors which fulfils $\mathbf{e}_1^{m_1} \ldots \mathbf{e}_r^{m_r} \mathbf{e}_{r+1}^{m_{r+1}} \ldots \mathbf{e}_n^{m_n} = -\mathbf{e}_1^{m_1} \ldots \mathbf{e}_{r+1}^{m_{r+1}} \mathbf{e}_r^{m_r} \ldots \mathbf{e}_n^{m_n} (m_r = m_{r+1} = 1)$. The element $I = \mathbf{e}_1 \ldots \mathbf{e}_n$ is called unit pseudoscalar and any pseudoscalar can be written as $P = \alpha I$ where α is a scalar. If I^{-1} denotes the inverse of I, so that $II^{-1} = 1$, then the magnitude of P relative to I will be called **bracket** and is defined by $PI^{-1} = \alpha \equiv [P]$. The bracket of the maximum grade multivector $\boldsymbol{a}_1 \ldots \boldsymbol{a}_n$ is determined by $[\boldsymbol{a}_1 \ldots \boldsymbol{a}_n] \equiv [\boldsymbol{a}_1 \wedge \boldsymbol{a}_2 \wedge \ldots \wedge \boldsymbol{a}_n] = (\boldsymbol{a}_1 \wedge \boldsymbol{a}_2 \wedge \ldots \wedge \boldsymbol{a}_n) I^{-1}$ and can be taken as a definition of the determinant. The dual $\boldsymbol{A}^*$ of an r-vector $\boldsymbol{A}$ is defined by $\boldsymbol{A}^* = \boldsymbol{A} I^{-1}$. In a geometric algebra of an n-dimensional vector space one can define the **join** for the linearly independent r-vector $\boldsymbol{A}$ and s-vector $\boldsymbol{B}$ by $\boldsymbol{J} = \boldsymbol{A} \wedge \boldsymbol{B}$. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are not linearly independent the join is not given simply by the wedge but by the commonly spanned subspace. If $\boldsymbol{A}$ and $\boldsymbol{B}$ have a common factor (i.e. there exists a k-vector $\boldsymbol{C}$ such that $\boldsymbol{A} = \boldsymbol{A}'\boldsymbol{C}$ and $\boldsymbol{B} = \boldsymbol{B}'\boldsymbol{C}$ for some $\boldsymbol{A}'$, $\boldsymbol{B}'$) then we can define the 'intersection' or **meet** of $\boldsymbol{A}$ and $\boldsymbol{B}$ as $\boldsymbol{A} \vee \boldsymbol{B}$ where $(\boldsymbol{A} \vee \boldsymbol{B})^* = \boldsymbol{A}^* \wedge \boldsymbol{B}^*$. Thus the dual of the meet is given by the join of the duals.

The paper is organized as follows: Section two presents a new method for computing the intrinsic camera parameters based on Pascal's theorem. Section three is devoted to the experimental analysis and section four to the conclusion part.

2 Camera Calibration using Pascal's Theorem

This section presents a new technique in the geometric algebra framework for computing the intrinsic camera parameters. In this context we use the ideas of Maybank, Faugeras and Luong [5, 4] to enforce an epipolar line defined by the epipole or fundamental matrix and a point $(1, \tau, 0)^T$ to be tangential to the image of the absolute conic and analyse Pascal's theorem in this context. Consider the points $\boldsymbol{a}$, $\boldsymbol{a}'$, $\boldsymbol{b}$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ lying on a conic, we can compute the following bracket expression

$$[\boldsymbol{c}'\boldsymbol{a}\boldsymbol{b}][\boldsymbol{c}'\boldsymbol{a}'\boldsymbol{b}'] - \rho[\boldsymbol{c}'\boldsymbol{a}\boldsymbol{b}'][\boldsymbol{c}'\boldsymbol{a}'\boldsymbol{b}] = 0 \qquad \Rightarrow \rho = \frac{[\boldsymbol{c}'\boldsymbol{a}\boldsymbol{b}][\boldsymbol{c}'\boldsymbol{a}'\boldsymbol{b}']}{[\boldsymbol{c}'\boldsymbol{a}\boldsymbol{b}'][\boldsymbol{c}'\boldsymbol{a}'\boldsymbol{b}]} \tag{1}$$

for some $\rho \neq 0$. This equation is well known and represents a projective invariant which has been used often in real applications of computer vision. Now consider $\boldsymbol{c}$ as some other point placed on the conic, we can get a conic equation fully represented in terms of brackets

$$[\boldsymbol{c}\boldsymbol{a}\boldsymbol{b}][\boldsymbol{c}\boldsymbol{a}'\boldsymbol{b}'][\boldsymbol{a}\boldsymbol{b}'\boldsymbol{c}'][\boldsymbol{a}'\boldsymbol{b}\boldsymbol{c}'] - [\boldsymbol{c}\boldsymbol{a}\boldsymbol{b}'][\boldsymbol{c}\boldsymbol{a}'\boldsymbol{b}][\boldsymbol{a}\boldsymbol{b}\boldsymbol{c}'][\boldsymbol{a}'\boldsymbol{b}'\boldsymbol{c}'] = 0. \tag{2}$$

For further details about describing a conic by brackets and points placed on the conic the reader should consult [2]. Using the collinearity constraint, we can write the geometric formulation of Pascal's theorem

$$\underbrace{((\boldsymbol{a}' \wedge \boldsymbol{b}) \vee (\boldsymbol{c}' \wedge \boldsymbol{c}))}_{\boldsymbol{\alpha}_1} \wedge \underbrace{((\boldsymbol{a}' \wedge \boldsymbol{a}) \vee (\boldsymbol{b}' \wedge \boldsymbol{c}))}_{\boldsymbol{\alpha}_2} \wedge \underbrace{((\boldsymbol{c}' \wedge \boldsymbol{a}) \vee (\boldsymbol{b}' \wedge \boldsymbol{b}))}_{\boldsymbol{\alpha}_3} = 0. \tag{3}$$

This theorem proves that the three intersecting points of the lines which connect opposite vertices of a hexagon circumscribed by a conic are collinear ones. Since equation (3) fulfills a property of any conic, it should also be possible to compute the intrinsic camera parameters using this equation. In geometric algebra a conic can be described by the points lying on the conic. Furthermore, the image of the absolute conic can be described by the image of the points lying on the absolute conic. Let us choose for all computations the following imaginary points lying on the absolute conic

$$\boldsymbol{A}_0 = \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix}, \boldsymbol{B}_0 = \begin{pmatrix} i \\ 1 \\ 0 \\ 0 \end{pmatrix}, \boldsymbol{A}'_0 = \begin{pmatrix} i \\ 0 \\ 1 \\ 0 \end{pmatrix}, \boldsymbol{B}'_0 = \begin{pmatrix} 1 \\ 0 \\ i \\ 0 \end{pmatrix}, \boldsymbol{C}'_0 = \begin{pmatrix} 0 \\ i \\ 1 \\ 0 \end{pmatrix}, \tag{4}$$

where $i^2 = -1$. The projected points of $\boldsymbol{A}_0, \boldsymbol{B}_0, \boldsymbol{A}'_0, \boldsymbol{B}'_0, \boldsymbol{C}'_0$ can be described by $\boldsymbol{a} = K[R|\boldsymbol{t}]\boldsymbol{A}_0 = KR\boldsymbol{A}, \boldsymbol{b} = K[R|\boldsymbol{t}]\boldsymbol{B}_0 = KR\boldsymbol{B}, \boldsymbol{a}' = K[R|\boldsymbol{t}]\boldsymbol{A}'_0 = KR\boldsymbol{A}', \boldsymbol{b}' = K[R|\boldsymbol{t}]\boldsymbol{B}'_0 = KR\boldsymbol{B}', \boldsymbol{c}' = K[R|\boldsymbol{t}]\boldsymbol{C}'_0 = KR\boldsymbol{C}'$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix}, \boldsymbol{A}' = \begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix}, \boldsymbol{B}' = \begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix}, \boldsymbol{C}' = \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix}. \tag{5}$$

In addition the rotated points $R^T\boldsymbol{A}, \ldots, R^T\boldsymbol{C}'$ also lie at the conic. Using the rotated points, the image of the absolute conic can be described by $\boldsymbol{a} = K\boldsymbol{A}, \boldsymbol{b} = K\boldsymbol{b}, \boldsymbol{a}' = K\boldsymbol{A}', \boldsymbol{b}' = K\boldsymbol{b}', \boldsymbol{c}' = K\boldsymbol{C}'$. To use $\boldsymbol{a}, \ldots, \boldsymbol{c}'$ in the bracket notation of conics we suppose an orthonormal basis of the image plane, $B_1 = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and formulate $\boldsymbol{X}$ as a linear combination of B_1, i.e. $\boldsymbol{X} = \sum_{i=1}^{3} x_i \mathbf{e}_i$ and introduce an operator $\underline{K}\mathbf{e}_i = \underline{K}(\mathbf{e}_i) = \sum_{j=1}^{3} \mathbf{e}_j k_{ji}$, with k_{ji} the elements of the matrix of intrinsic camera parameters, K.

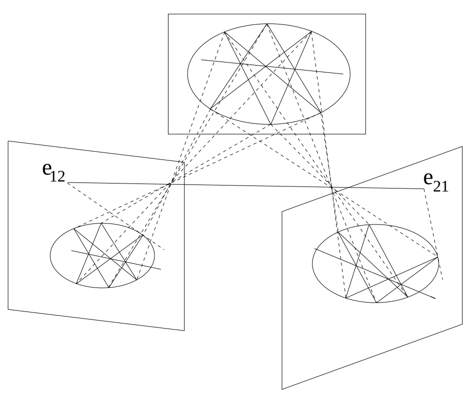

Fig. 1. Pascal's theorem in the conic images

The application of the principle of duality allows us to set $\boldsymbol{c} = KK^T\boldsymbol{l}_c$ for a point $\boldsymbol{c}$ on the image of the absolute conic. This point depends on the intrinsic camera parameters and a line $\boldsymbol{l}_c$ tangent to the image of the absolute conic computed in terms of the epipole and a point $\boldsymbol{q}$ lying at infinity, $\boldsymbol{l}_c =$

$(\boldsymbol{e}_{12} \wedge \boldsymbol{q})\boldsymbol{I}^{-1} = (p_1\mathrm{e}_1 + p_2\mathrm{e}_2 + p_3\mathrm{e}_3) \wedge (\mathrm{e}_1 + \tau\mathrm{e}_2)\boldsymbol{I}^{-1}$. The point $\boldsymbol{c}$ can be described by using the adjoint operator $\overline{K}$ of $\underline{K}$. The above expression for the point $\boldsymbol{c}$ can thus be formulated as $\boldsymbol{c} = \underline{K}\overline{K}\boldsymbol{l}_c$.

Now we simplify equation (3) and get the bracket equation of the $\boldsymbol{\alpha}$'s

$$\begin{aligned}
&\big([\boldsymbol{a}'\boldsymbol{b}\boldsymbol{c}']\boldsymbol{c} - [\boldsymbol{a}'\boldsymbol{b}\boldsymbol{c}]\boldsymbol{c}'\big) \wedge \big([\boldsymbol{a}'\boldsymbol{a}\boldsymbol{b}']\boldsymbol{c} - [\boldsymbol{a}'\boldsymbol{a}\boldsymbol{c}]\boldsymbol{b}'\big) \wedge \big([\boldsymbol{c}'\boldsymbol{a}\boldsymbol{b}']\boldsymbol{b} - [\boldsymbol{c}'\boldsymbol{a}\boldsymbol{b}]\boldsymbol{b}'\big) = 0 \\
\Leftrightarrow\; &\underbrace{\big([\boldsymbol{A}'\boldsymbol{B}\boldsymbol{C}'](\overline{K}\boldsymbol{l}_c) - [\boldsymbol{A}'\boldsymbol{B}(\overline{K}\boldsymbol{l}_c)]\boldsymbol{C}'\big)}_{\boldsymbol{\alpha}_1} \wedge \underbrace{\big([\boldsymbol{A}'\boldsymbol{A}\boldsymbol{B}'](\overline{K}\boldsymbol{l}_c) - [\boldsymbol{A}'\boldsymbol{A}(\overline{K}\boldsymbol{l}_c)]\boldsymbol{B}'\big)}_{\boldsymbol{\alpha}_2} \wedge \\
&\underbrace{\big([\boldsymbol{C}'\boldsymbol{A}\boldsymbol{B}']\boldsymbol{B} - [\boldsymbol{C}'\boldsymbol{A}\boldsymbol{B}]\boldsymbol{B}'\big)}_{\boldsymbol{\alpha}_3} = 0.
\end{aligned} \tag{6}$$

The computation of the intrinsic parameters is done first if the intrinsic parameters remain stationary under camera motions and second if these parameters change.

2.1 Computing stationary intrinsic parameters

We consider one camera motion. The involved projective transformations are $P_1 = K[I|0]$ and $P_2 = P_1 \begin{pmatrix} R_1 & \boldsymbol{t}_1 \\ 0_3^T & 1 \end{pmatrix}^{-1} = P_1 D^{-1}$. The 4×4-matrix D describes the extrinsic camera parameters. The optical centres of the cameras are $\boldsymbol{C}_1 = (0,0,0,1)^T$ and $\boldsymbol{C}_2 = D\boldsymbol{C}_1$. In geometric algebra we use instead the notations $\underline{P_1}$, $\underline{P_2}$, $\boldsymbol{C}_1 = \mathrm{e}_4$ and $\boldsymbol{C}_2 = \underline{D}\boldsymbol{C}_1$. Thus we can compute their epipoles as $\boldsymbol{e}_{21} = \underline{P_2}\boldsymbol{C}_1$, $\boldsymbol{e}_{12} = \underline{P_1}\boldsymbol{C}_2$.

Next, we show by means of an example that the homogeneous coordinates of the points $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\alpha}_3$ are entirely independent of the intrinsic parameters. This condition is necessary for solving the problem. We choose a camera motion given by

$$[R_1|\boldsymbol{t}_1] = \left(\begin{array}{ccc|c} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right). \tag{7}$$

For this motion the epipoles are $\boldsymbol{e}_{12} = (2k_{11} - k_{12} + 3k_{13})\mathrm{e}_1 + (-k_{22} + 3k_{23})\mathrm{e}_2 + 3\mathrm{e}_3$ and $\boldsymbol{e}_{21} = (k_{11} + 2k_{12} - 3k_{13})\mathrm{e}_1 + (2k_{22} - 3k_{23})\mathrm{e}_2 - 3\mathrm{e}_3$. By replacing $\boldsymbol{e}_{12}$ in the equation (6), we can make explicit the $\boldsymbol{\alpha}'s$

$$\begin{aligned}
\boldsymbol{\alpha}_1 &= ((-3+3i)k_{11}\tau)\mathrm{e}_1 + (3k_{11}\tau - ik_{12}\tau + ik_{22} + 2ik_{11}\tau - 3k_{12}\tau + 3k_{22})\mathrm{e}_2 + \\
&\quad (ik_{11}\tau + 3k_{12}\tau - 3k_{22} + ik_{12}\tau - ik_{22})\mathrm{e}_3 \\
\boldsymbol{\alpha}_2 &= (-3ik_{11}\tau - 2k_{12}\tau + 2k_{22} - 2k_{11}\tau)\mathrm{e}_1 + (-6i(k_{12}\tau - k_{22}))\mathrm{e}_2 + \\
&\quad (-3k_{11}\tau - 4ik_{12}\tau + 4ik_{22} + 2ik_{11}\tau)\mathrm{e}_3 \\
\boldsymbol{\alpha}_3 &= (1-i)\mathrm{e}_1 + (1-i)\mathrm{e}_2 + 2\mathrm{e}_3.
\end{aligned} \tag{8}$$

Note that $\boldsymbol{\alpha}_3$ is fully independent of $\underline{K}$.

According to Pascal's theorem these three points are lying on the same line. Thereto, by replacing these points in Pascal's equation (3), we get the following second order polynomial in τ

$$(-40ik_{12}^2 - 52ik_{11}^2 + 16ik_{11}k_{12})\tau^2 + (-16ik_{11}k_{22} + 80ik_{12}k_{22})\tau - 40ik_{22}^2 = 0. \tag{9}$$

Solving this polynomial and choosing one of the solutions which is nothing else than the solution for one of the two lines tangent to the conic, we get

$$\tau := \frac{16ik_{11}k_{22} - 80ik_{12}k_{22} + 24\sqrt{14}k_{11}k_{22}}{2(-40ik_{12}^2 - 52ik_{11}^2 + 16ik_{11}k_{12})}. \tag{10}$$

Now considering the homogeneous representation of the intersecting points

$$\boldsymbol{\alpha}_i = \alpha_{i1}\mathbf{e}_1 + \alpha_{i2}\mathbf{e}_2 + \alpha_{i3}\mathbf{e}_3 \sim \frac{\alpha_{i1}}{\alpha_{i3}}\mathbf{e}_1 + \frac{\alpha_{i2}}{\alpha_{i3}}\mathbf{e}_2 + \mathbf{e}_3, \tag{11}$$

and the case of exactly orthogonal image axis, $k_{12} = 0$, we get

$$\begin{aligned} \alpha_{11} &= \frac{2i - 3\sqrt{14} + 10 + 2i\sqrt{14}}{2 + 3i\sqrt{14} + 16i + 2\sqrt{14}} \quad \alpha_{12} = 26\frac{i}{2 + 3i\sqrt{14} + 16i + 2\sqrt{14}} \\ \alpha_{21} &= \frac{(1+i)(-2i + 3\sqrt{14})}{-5i + \sqrt{14} - 13} \quad \alpha_{22} = -\frac{-11 + 3i\sqrt{14} - 3i - 2\sqrt{14}}{-5i + \sqrt{14} - 13}. \end{aligned} \tag{12}$$

The coordinates are indeed independent of the intrinsic parameters. After this illustration which helps for the understanding we will get now the coordinates using any camera motion. For that let us define $\boldsymbol{s} = s_1\mathbf{e}_1 + s_2\mathbf{e}_2 + s_3\mathbf{e}_3 = \underline{[I|0]}\,\underline{D}\boldsymbol{C}_1$. Using this value, the epipole is $\boldsymbol{e}_{12} = \underline{K}\,\underline{[I|0]}\,\underline{D}\boldsymbol{C}_1 = \underline{K}\boldsymbol{s}$. Note that in this expression the intrinsic camera parameters are separate from the extrinsic ones. Similar as above using the general camera motion and the epipole value, the coordinates for the intersecting points read

$$\begin{aligned} \alpha_{11} &= -\frac{(-s_3s_1s_2 + is_3\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} - is_3^3 - is_3s_1^2 + s_1\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} - is_2s_3^2)}{(-is_3s_1s_2 - s_3\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} - s_3^3 - s_3s_1^2 + is_1\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} + s_2s_3^2)} \\ \alpha_{21} &= \frac{-2s_3(s_3^2+s_1^2)}{-is_3s_2s_1 - s_3\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} - s_3^3 - s_3s_1^2 + is_1\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} + s_2s_3^2} \\ \alpha_{12} &= \frac{(-1-i)(is_1s_2 + \sqrt{s_3^2(s_1^2+s_2^2+s_3^2)})s_3}{-is_3s_1s_2 - s_3\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} + s_3s_1^2 + s_3^3 + s_1\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} - is_2s_3^2} \\ \alpha_{22} &= \frac{i(is_3s_1s_2 + s_3\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} + is_1\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} + s_2s_3^2 + is_3s_1^2 + is_3^3)}{-is_3s_1s_2 - s_3\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} + s_3s_1^2 + s_3^3 + s_1\sqrt{s_3^2(s_1^2+s_2^2+s_3^2)} - is_2s_3^2}. \end{aligned} \tag{13}$$

Note that also in the general case the intrinsic parameters are totally cancelled out. These invariance properties can be used to obtain equations which depend on the four unknown intrinsic camera parameters. For this we suppose point correspondences between two cameras and the motion parameters. First, the values of the invariant homogeneous $\boldsymbol{\alpha}_i$ can be calculated by the known motion and formulas (13). Second, we calculate the epipole from the point correspondences, calculate $\overline{K}\boldsymbol{l}_c$ and solve an equation system for τ similar to (10) to achieve a polynomial depending on the intrinsic camera parameters which must be equal to the calculated values of the $\boldsymbol{\alpha}_i$'s by (13). Thus, we should find another set of equations to solve the problem. The way to do that is simply to consider the second camera with its epipole $\boldsymbol{e}_{21}$. Since we are assuming that the intrinsic parameters remain constant, we can consequently gain a second set of four equations depending again of the four intrinsic parameters from the second epipole.

The interesting aspect here is that we require only one camera motion to find a solvable equation system. Other methods gain for each camera motion only a couple of equations, thus they require at least three camera motions to solve the problem [4].

2.2 Computing non–stationary intrinsic parameters

As a difference we compute now the line $\boldsymbol{l}_c$ using the fundamental matrix and a point lying at infinity of the second camera which is equal to the cross product of the second epipole with the point at infinity, namely $\boldsymbol{l}_c = \overline{F}(\mathbf{e}_1 + \tau'\mathbf{e}_2)$.

Now similar as in previous case we will use an example for facilitating the understanding. We will use the same camera motion given in equation (7) and suppose orthogonal image axis.

Similar as above we compute the $\boldsymbol{\alpha}$'s and according Pascal's theorem we gain a polynomial similar as equation (9). We select one of both solutions of the gained second order polynomial and substitute it in the homogeneous coordinates of the $\boldsymbol{\alpha}$'s given by

$$\alpha_{11} = -\frac{i(-5i - 4 + i\sqrt{14})}{5i + 2 + 2i\sqrt{14}} \qquad \alpha_{21} = \frac{-2 + 3i\sqrt{14}}{5i + 2 + 2i\sqrt{14}} \tag{14}$$

$$\alpha_{12} = \frac{10 - 10i}{-4i - 2 + 3i\sqrt{14} - \sqrt{14}} \qquad \alpha_{22} = -\frac{8 + 6i - \sqrt{14} + 3i\sqrt{14}}{-4i - 2 + 3i\sqrt{14} - \sqrt{14}}. \tag{15}$$

Now we will consider the general motion

$$[R|\boldsymbol{t}] = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{pmatrix}. \tag{16}$$

In matrix algebra the fundamental matrix reads

$$F = K^{-T} E K'^{-1} = \begin{pmatrix} k_{11} & 0 & k_{13} \\ 0 & k_{22} & k_{23} \\ 0 & 0 & 1 \end{pmatrix}^{-T} \begin{pmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{pmatrix} \begin{pmatrix} k'_{11} & 0 & k'_{13} \\ 0 & k'_{22} & k'_{23} \\ 0 & 0 & 1 \end{pmatrix}^{-1} \tag{17}$$

and in geometric algebra the operator of the fundamental matrix reads $\underline{F} = \overline{K}^{-1} \underline{E}\, \underline{K}'^{-1}$. The matrix $E = [\boldsymbol{t}]_\times R_1$ is the so called essential matrix. We can compute the $\boldsymbol{\alpha}$'s using this general expression for $\underline{F}$ and get again equations fully independent of the intrinsic parameters. Together with the equations of the $\boldsymbol{\alpha}$'s obtained using the first epipole, the intrinsic parameters can be found by solving a quadratic equation system.

3 Experimental Analysis

In this section we present the test of the method based on Pascal's theorem using firstly simulated images. We will explore the effect of increasing noise in the computation of the intrinsic camera parameters. The experiments with real images show that the performance of the method is reliable.

3.1 Experiments with simulated images

In order to test the performance of our approach we carried out a motion of the camera about the y–axis and a small translation along the three camera axes by increasing noise. For the tests we used exact arithmetic instead of floating point arithmetic. The Table 1 shows the computed intrinsic parameters. The most right column of the table shows the value obtained by substituting these parameters in the polynomial (9) which gives zero for the case of zero noise. The values in this column show that by increasing noise the computed intrinsic parameters cause a tiny deviation of the ideal value of zero. This indicates that the procedure is relatively stable against noise. Note that there are remarkable deviations shown by noise 1.25.

Noise(pixels)	k_{11}	k_{13}	k_{22}	k_{23}	Error
0	500	256	500	256	10^{-8}
0.5	504	259.5	503.5	258	0.004897
1	482	242	485	254	0.011517
1.25	473	220	440	238	0.031206
1.5	517	272	518	266	0.015
2	508	262.5	504	258.5	0.006114

Table 1. Intrinsic parameters by rotation about the y–axis and translation along the three axes by increasing noise.

3.2 Experiments with real images

In this section we present experiments using real images with one general camera motion as shown in Figure 2. The motion was done about the three coordinate axes. We used a calibration dice and for comparison purposes we computed the intrinsic parameters from the involved projective matrices by splitting the intrinsic parameters from the extrinsic ones. The reference values were: First camera $k_{11} = 1200.66$, $k_{22} = 1154.77$, $k_{13} = 424.49$, $k_{23} = 264.389$ and second camera $k_{11} = 1187.82$, $k_{22} = 1141.58$, $k_{13} = 386.797$, $k_{23} = 288.492$ with mean errors of 0.688 and 0.494, respectively. Thereafter using the gained parameters $[R_1|\boldsymbol{t}_1]$ and $[R_2|\boldsymbol{t}_2]$ we computed the $[R|\boldsymbol{t}]$ between cameras which is required for the Pascal's theorem based method. The fundamental matrices were computed using a non-linear method. Using the method of Pascal's theorem with 12 point correspondences unlike 160 point correspondences used by the algorithm with the calibration dice we computed the following intrinsic parameters $k_{11} = 1244$, $k_{22} = 1167$, $k_{13} = 462$ and $k_{23} = 217$. These values resemble quite well to the reference ones and cause an error of $\sqrt{|eqn_1|^2 + ... + |eqn_8|^2}$: 0.00496045 in the error function, where eqn_i are the constraint equations depending on the intrinsic parameters. The difference to the reference values is attributable to inherent noise in the computation and to the fact that the reference values are not exact, too.

Since a system of quadratic equations is to be solved we resort to an iterative procedure for finding the solution. First, we tried the *Newton–Raphson method* and the *Continuation method* [4]. These methods were not practicable enough due to their complexity. We used instead a variable in size window minima search which through the computation ensures the reduction of the quadratic error. This simply approach worked faster and reliable.

In order to visualize how good we gain the epipolar geometry we superimposed the epipolar lines for some points using the reference method and method of Pascal's theorem. In both cases we computed the fundamental matrix in terms of their intrinsic parameters, i.e. $F = K^{-T}([\boldsymbol{t}]_\times R)K^{-1}$. Figure 2.r shows this comparison. It is clear that both methods give quite similar epipolar lines and interesting enough it is shown that the intersecting point or epipole coincides almost exactly.

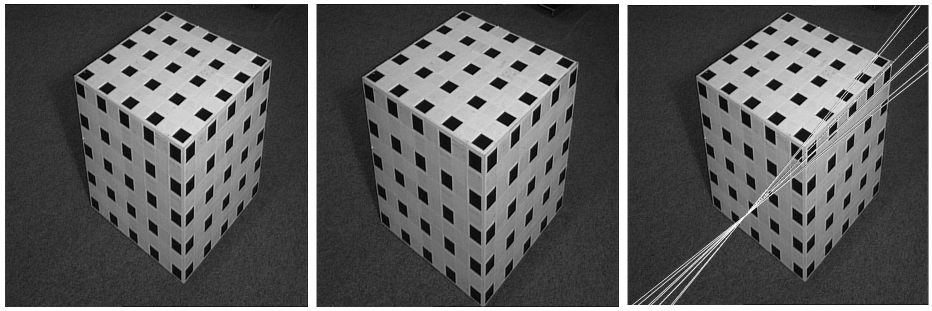

Fig. 2. Scenario and (r.) superimposed epipolar lines using the reference and Pascal's theorem based method.

4 Conclusions

This paper presents a geometric approach to compute the intrinsic camera parameters in the geometric algebra framework using Pascal's theorem. We adopted the projected characteristics of the absolute conic in terms of Pascal's theorem to propose a new camera calibration method based on geometric thoughts. The use of this theorem in the geometric algebra framework allows us the computation of a projective invariant using the conics of only two images. Then, this projective invariant expressed in terms of brackets helps us to set enough equations to solve the calibration problem. Our method requires to know the point correspondences and the values of the camera motion. The method gives a new point of view for the understanding of the problem thanks to the application of Pascal's theorem and it also explains the overseen role of the projective invariant in terms of the brackets. Using synthetic and real images we show that the method performs efficiently without any initialization or getting trapped in local minima.

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