

CHANGING THE SAMPLING RATE OF VIDEO SIGNALS BY RATIONAL FACTORS

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An FIR-filter design is presented for changing the sampling rate of video signals by rational factors. Most of the widely used computer programs utilize an equiripple design for passband and stopband. A nonuniform weighting function with equispaced zeros in the stopband is proposed for FIR-filters of sampling rate converters. This allows a significant reduction of the number of taps under the constraint that no visible impairments are produced. The filter design is performed by mixed-integer optimization to provide finite precision coefficients. A floating point representation of the coefficients with a 4-bit mantissa is advantageous and reduces the cost of hardware implementation.

INTRODUCTION

There have been discussions on digital coding standards for broadcast television signals within the committees of EBU, SMPTE, and CCIR. As a result of these discussions for digital studios component coding with a sampling rate of 13.5 MHz for the luminance component and 6.75 MHz for the chrominance signals was proposed /1/. Application of these sampling rates results in the very high transmission rate of 216 Mbit/sec for digital TV signals. Besides the studio quality level there are other levels like electronic news gathering (ENG level) and an auxiliary level which do not require such high sampling rates. For distribution services like cable TV, or video conferencing lower sampling rates are possible as well.

It is desired that the new sampling rates for lower quality levels could easily be derived from the studio standard. For this reason a conversion of the sampling rates by rational factors L/M is proposed, e.g. $L/M = 3/4$ changes the sampling rate of 13.5 MHz to 10.125 MHz. This paper is concerned with an FIR-filter design for sampling rate converters. There is a tutorial paper on this topic /2/ but the present paper is concerned with a design under consideration of the special features of video signals.

BASIC CONCEPT OF SAMPLING RATE CONVERSION

A principal description of a sampling rate conversion by rational factors is given in Figure 1. The sampling rate f_{cx} of the input signal $x(n)$ is first increased by a factor L by insertion of zeros. The zeros of the new sequence $u(n)$ are then interpolated by an FIR-filter. The convolution of the sequence $u(n)$ with the impulse response $h(n)$ results in the sequence $v(n)$. A subsampling $1:M$ of the sequence $v(n)$ provides the desired sequence $y(n)$ with the sampling rate $f_{cy} = f_{cx} * L/M$.

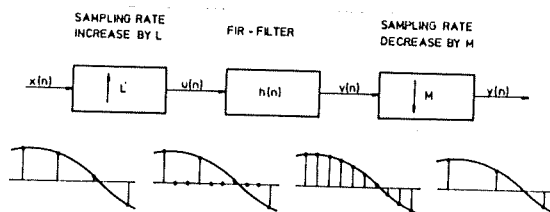


Fig.1 Block diagram and typical waveforms of sampling rate conversion.

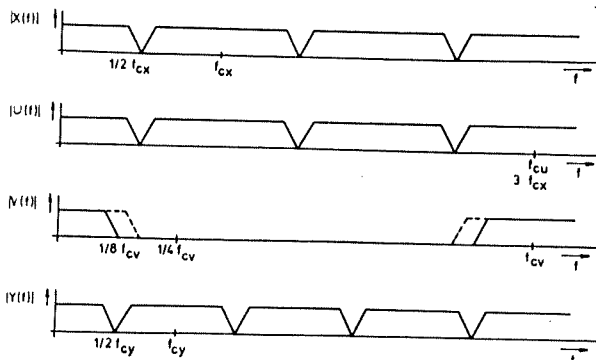


Fig.2 Spectral distribution for a sampling rate change by factor of 3/4.

The spectra of the signals are shown in Figure 2. The increase of sampling rate by insertion of zeros does not change the spectra but the sampling rate. The interpolation is provided by a low pass filter which has to remove the undesired images of the baseband spectrum. In the frequency domain the subsampling $1:M$ of $v(n)$ results in a superposition of shifted spectra of $V(f)$. Hereby the spectrum $V(f)$ has to be shifted by a multiple of f_{cy}/M . Thus for $L/M < 1$ the low pass filter has to reduce

the bandwidth such that aliasing is avoided.

Considering that only every L'th sample of $u(n)$ is different from zero and only every M'th sample of $v(n)$ is required for the output $y(n)$ a more appropriate realization in polyphase structure can be derived /2/. This kind of polyphase filter consists of L parallel filters. The set of coefficients of the polyphase filters are given by $g_r(n)$ where each $g_r(n)$ represents a subset of the impulse response $h(n)$ such that

$$g_r(n) = h(nL+r) \quad r=0, \dots, L-1. \quad (1)$$

The polyphase structure is very advantageous for video applications because each of the parallel filters has to provide an output value only for every Lth output sample. This gives relaxed conditions for hardware realization by a conventional pipeline technique.

DESIGN OF THE FILTER CHARACTERISTIC

The FIR-filter of the sampling rate converter has to provide a small interpolation error and, in addition, distortions like aliasing should be avoided. In order to keep the implementation costs as low as possible it is desired to know how many filter coefficients are required and how many bits are necessary to represent these coefficients. For video signals subjective picture quality after sampling rate conversion should be the optimization criterion. Because of the problem of incorporating subjective criteria in the design procedure the filter design is carried out in the frequency domain by known approximation programs. A sampling rate conversion of real television scenes with the resulting filter is accomplished by non-real-time processing on a computer. After processing the television scenes are displayed on a monitor to visualize the distortions introduced by sampling rate conversion.

As an example, in this paper results for a factor $L/M = 3/4$ are discussed which converts the luminance sampling frequency from 13.5 MHz down to 10.125 MHz. Further it is assumed that for this case a passband edge frequency of about 4 MHz is necessary.

The filter for sampling rate conversion are designed by Chebyshev approximation methods. The desired ideal frequency response is given by

$$D(f) = \begin{cases} L & 0 \leq f \leq f_p \\ 0 & f_s \leq f \leq 1/2 \end{cases} \quad (2)$$

where f is the normalized frequency and f_p is the passband edge frequency and f_s is the stopband edge frequency. The nominal stopband edge in order to avoid aliasing is given by

$$f_s = \begin{cases} 1/2L & L/M \geq 1 \\ 1/2M & L/M < 1. \end{cases} \quad (3)$$

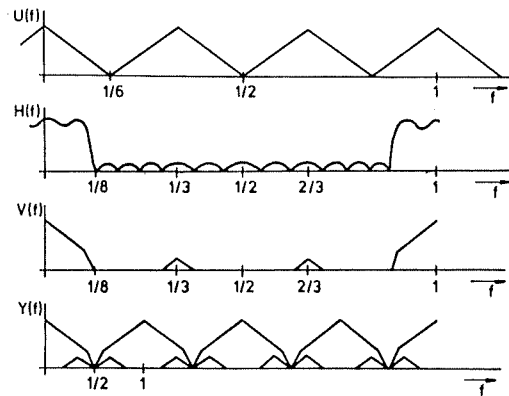


Fig.3 Generation of aliasing in case of insufficient stopband attenuation.

Based on this edge frequencies $f_p = 0.1$ and $f_s = 0.125$ are used for $L/M = 3/4$. FIR-filters have been designed for various number of taps N by means of a widely used computer program /3/. To avoid visible distortions even for critical pictures a stopband attenuation of about 40 dB is necessary. This requires a filter with $N = 81$. A hardware realization of such a high order filter would be very expensive for the given sampling frequencies, thus we looked for a design which allows a reduction in the number of taps N .

In case of insufficient attenuation the visible distortions are caused by low frequency components of the input signal $x(n)$, as it was derived from an analysis of the sampling rate conversion. This is illustrated in Figure 3. Video signal spectra of natural pictures concentrate most of their energy in low frequency components. The filtering of the intermediate sequence $v(n)$ could be summarized such that on one hand the high frequency components in the baseband are reduced, on the other hand in case of insufficient attenuation there remain undesired spectral components at frequencies $k * (1/L)$. The subsampling $1:M$ shifts the undesired spectra into the passband and this produces visible distortions. Figure 4 shows these distortions for a test picture. Figure 4b is the resulting picture after sampling rate conversion with a filter of order $N = 21$ (stopband attenuation about 15 dB). The horizontal picture size is reduced by a factor of $3/4$ because both pictures are displayed with the same clock frequency. Distortions are shown in Figure 4b can be essentially reduced by designing FIR-filters with the additional constraint that zeros are forced at the critical frequencies $k * (1/L)$. A sampling rate converter provides no distortions in areas of constant input signal if

$$H(k/L) = \begin{cases} L & k=0 \\ 0 & k=1, \dots, [L/2] \end{cases} \quad (4)$$



Fig.4 a.Original test picture.
b.Test picture after sampling rate conversion by a factor of 3/4. An FIR-filter with $N = 21$ and a stopband attenuation of 15 dB has been used.

It is easy to show that the preceding condition is equivalent to

$$\sum_j h(jL+r) = 1 \quad r=0, \dots, L-1 \quad (5)$$

which means that the sum of the coefficients in each path of the polyphase filter is equal to one.

For simplification of hardware realization the filter coefficients are modified to a quotient of an integer numerator $p(n)$ and a common denominator, which is a power of two. As shown in /4/ it is much more efficient to design FIR-filters under consideration of a specific integer representation than to design coefficients with infinite precision and rounding them afterwards to a desired precision. For this reason a special filter design program with mixed integer optimization for sampling rate converters has been developed. The design program considers the special constraints given by (5). The optimization algorithm used in this program is based on that given in /5/. The hardware implementation costs depend strongly on the ex-

pense for multipliers. For this reason the coefficients are further modified.

$$h(n) \rightarrow h'(n) = \frac{p(n) 2^{s(n)}}{2^B} \quad (6)$$

The numerator in (6) could be described as a floating point representation. The samples of video signals are coded by 8 bit. Our investigations have shown that for the power of the denominator $B = 8$ is sufficient. The number of bits for $p(n)$ is 4 bit in case of resolution charts and 2 bit in case of natural test pictures. Correspondingly, $s(n)$ is in a range 0...4 and 0...6 respectively. A hardware realization of video filters requires separate multipliers for each product. In such a case the shift operation by $s(n)$ can be hardwired and thus no additional hardware is required.

The visibility of distortions in pictures depends on their spatial frequency. The visual sensitivity decreases strongly for frequencies close to the passband edge. In addition distortions occurring at or adjacent to large signal changes are less visible than errors of the same amplitude occurring in flat areas of a picture. Considering these perceptual effects aliasing is less visible in the presence of an original signal of larger amplitude. Due to this fact aliasing produced by a filter with extended transition between passband and stopband is not visible for most of the test pictures. For this reason the stopband edge frequency $f_s = 0.125$ has been modified to 0.15. FIR-filters with an extended transition require much lower filter length N for the same stopband attenuation.

The conditions given by (4) and (5) respectively were introduced to avoid aliasing effects in areas of constant video signal. A further improvement is possible if aliasing effects are reduced for all low frequency components. Hence, the stopband attenuation have to be increased in a larger surrounding of the critical frequencies $k * (1/L)$. This could be performed with a nonuniform weighting function in the stopband. One of the used weighting functions is given by

$$W(f) = 1 - \cos^2(\pi Lf) \quad f_s \leq f \leq 1/2 \quad (7)$$

Our investigations have shown that by combining all the proposals given above a filter length $N = 21$ is sufficient. The frequency characteristic of such a filter is drawn in Figure 5. The pertaining coefficients are listed in Table 1. Figure 6 shows the resulting picture quality after sampling rate change for a resolution chart. In case of resolution charts moire pattern become just visible if a down sampling by 3/4 and an up sampling by 4/3 is performed in sequence. To avoid any visible distortions for resolution charts a filter length $N = 27$ is necessary.

CONCLUSION

Sampling rate conversion for video signals has been investigated. Using standard FIR-filter design a filter length of $N = 81$ is necessary for a sampling rate change by a factor of $L/M = 3/4$. Various proposals are derived that allow a reduction of the filter length. Aliasing effects of low frequency components are reduced by introducing an additional constraint for filter design. Under consideration of the visibility of distortions produced by sampling rate conversion a filter with $N = 21$ is sufficient for $L/M = 3/4$. The described strategy can also be used to design filters for other conversion factors.

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	L/M=3/4	L/M=4/3
$h(0) = h(20)$	16/256	16/256
$h(1) = h(19)$	12/256	13/256
$h(2) = h(18)$	11/256	7/256
$h(3) = h(17)$	-28/256	-9/256
$h(4) = h(16)$	-36/256	-64/256
$h(5) = h(15)$	-40/256	-52/256
$h(6) = h(14)$	1/256	1/256
$h(7) = h(13)$	56/256	64/256
$h(8) = h(12)$	120/256	176/256
$h(9) = h(11)$	176/256	240/256
$h(10) =$	192/256	240/256

Table 1 Coefficients of the FIR-filter for sampling rate change with $L/M = 3/4$ and $L/M = 4/3$ respectively.

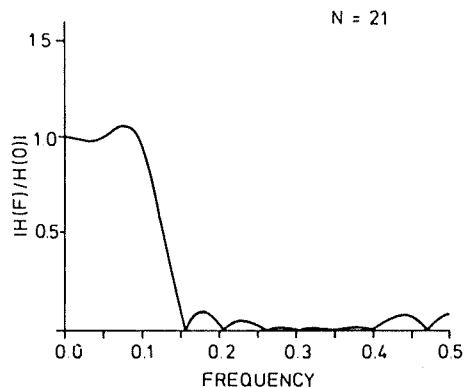


Fig.5 Frequency response of a filter for sampling rate change with $L/M = 3/4$ ($N = 21$)

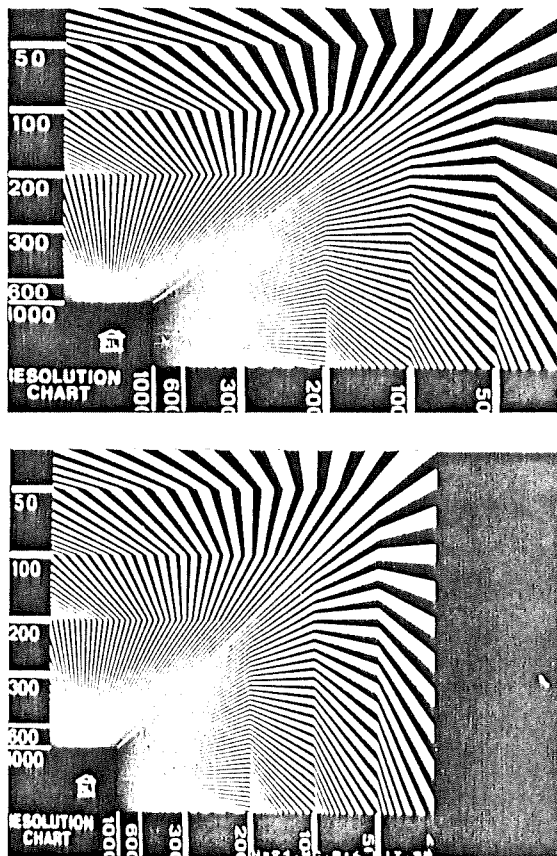


Fig.6 a. Original test picture
 b. Test picture after sampling rate conversion by a factor of 3/4. An FIR filter with a frequency characteristic as shown in Fig.5 is used.