Modified Active Constellation Extension Algorithm for PAPR Reduction in OFDM Systems

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***Abstract*—Communication systems based in Orthogonal Frequency Division Multiplexing (OFDM) technology are very popular due to their robustness against inter-symbol interference (ISI) and their efficient use of the spectrum. Nevertheless, one of the major drawbacks of OFDM is its high peak-to-average power ratio (PAPR), which is part of its multicarrier nature. A high PAPR could drive the high-power amplifier (HPA) in its nonlinear region, preventing thus the receiver from recovering the conveyed information correctly. To avoid this, a PAPR reduction algorithm is essential for such systems. Therefore, in this paper we introduce the modified active constellation extension (mACE) algorithm. The mACE capability to reduce the PAPR is demonstrated through simulation and compared with the state-of-the-art smart gradient-project (SGP) method. It is shown that mACE outperforms the SGP method. For instance, in systems with QPSK modulation, mACE reduces up to 0.5 dB more PAPR than SGP, and up to 0.2 dB in systems with 16-QAM. These results are achieved with less computational complexity. Hence, mACE achieves an appropriate trade-off between PAPR reduction and system resources, which makes it a viable option in real time OFDM systems.**

***Index Terms*—OFDM, PAPR, active constellation extension (ACE), modified ACE (mACE)**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is nowadays probably one of the most used multicarrier technologies in wireless and wireline communication standards. Among others well known technologies, for example, OFDM is the basis of Digital Audio Broadcasting (DAB) systems, Digital Video Broadcasting (DVB) systems, WiMAX, IEEE802.11, LTE and Bluethooth. Its popularity is due to its capability of dealing with multipath channels with a given delay spread, avoiding thus inter symbol interference (ISI) with a relative lower complexity in comparison with single carrier systems. However, one of the major drawbacks of OFDM systems is its inherently high Peak-to-Average Power Ratio (PAPR), which is proportional to the largest power peak in the transmitted signal in time-domain. Large peaks in the time-domain signal may provoke intermodulation products among subcarriers and a large out-of-band power since it makes the high power amplifier (HPA) work in its nonlinear region. Moreover, operation in the nonlinear region of HPA adds rotation, offset and attenuation to the transmitted signal, which distorts it in such a way that the receiver is incapable to demodulate it back. Therefore, linearity in HPA is a compulsory requirement for such systems, and it can only be achieved at the cost of efficiency, since efficiency and linearity are trade-off criteria for the design of HPAs. The reduction of PAPR is thus necessary for all kind of OFDM systems, however, it is essentially more important for small communication devices with limited resources as in mobile application systems.

Solving the PAPR problem is still an active area of research. In the attempts to mitigate the PAPR, many techniques have been proposed in the literature. Among a vast number of publications, some of these techniques are clipping techniques [1], [2], coding techniques [3], [4], [5], active constellation extension (ACE) [6], partial transmit sequence (PTS) [7]-[8], selected mapping (SLM)[8] [9] [10], interleaving [11] and DFT-spreading techniques [12]. A summary of some of these techniques can be found in [13]. Unfortunately, any attempt to reduce PAPR implies a trade-off with other system resources, i.e., the spectrum efficiency may be reduced while the transmission power, computing power and processing time could be increased. This means that the optimal solution suitable for all systems is still unknown and the technique to employ is rather system dependent.

In this paper we focus explicitly on the ACE algorithms, introduced in [6]. On the one hand, it is well known that ACE has a good performance without decreasing the bandwidth efficiency, on the other hand, it increases the signal power and the computing power. The increased power in the transmitter signal is a small drawback that is negligible in comparison to the operating consumption of a linear HPA. Nevertheless, the computational burden incurred by ACE for the PAPR reduction can be very high for systems with very limited computing resources. Therefore, we modify the ACE algorithm to decrease its computational complexity and, thus, also its delay. In this paper, we present a modification of the ACE algorithm and denote it as modified ACE (mACE). To this end, mACE is totally suitable for systems in which otherwise ACE would be implemented since mACE maintains the performance of ACE but requiring less resources. We find out that in some cases, mACE can even outperform ACE regarding its PAPR reduction capability but with a

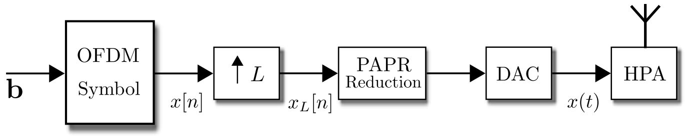

Fig. 1: OFDM PAPR reduction block diagram

lower complexity. This is proved by exhaustive simulation experiments that are presented in this paper.

The remainder of this paper is organized as follows. In Section II, an OFDM system is described while in Section III the original ACE algorithm is presented. In Section IV we introduce the modified ACE (mACE) algorithm for the PAPR reduction. Numerical results and performance comparisons for illustration are presented in Section V, followed by a conclusion and a summary of the main contributions of this paper in Section VI.

II. OFDM Systems

OFDM can be categorized as a multicarrier modulation technique [14] due to the fact that one OFDM symbol conveys N parallel data over the channel. The data symbols are allocated in orthogonal frequencies, denominated subcarriers, separated by $\Delta f = B/N = 1/T$, where B is the bandwidth of an OFDM symbol and T denotes its duration in time-domain. The orthogonality property in frequency domain allows a transmission without intercarrier interference (ICI) while ISI can be avoided by adding to it the so called cyclic prefix (CP) in the time domain. At the transmitter, therefore, the information bit vector $\mathbf{b} \in \{0,1\}^{1\times NM}$ is mapped into a complex value, information symbol vector $\mathbf{X} \in \mathbb{M}^{1\times N}$, where $\mathbb{M}$ is a fixed constellation set of M-QAM symbols and N is the total number of OFDM subcarriers. Afterwards, the frequency-domain OFDM symbol $\mathbf{X} = [X[k]]_{k=0}^{N-1}$ is transformed to its time-domain representation via the inverse discrete fourier transformation (IDFT) as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}, \tag{1}$$

with $j = \sqrt{-1}$. In this paper, we do not consider the CP, it is neither necessary for the PAPR computation nor for its reduction algorithm. Afterwards, the signal $\mathbf{x} = [x[n]]_{n=0}^{N-1}$ in (1) is converted to an analog signal $x(t)$ by means of the digital-to-analog converter (DAC), which may then be modulated into some higher (passband) frequency carrier. The complex, baseband signal $x(t)$ is finally amplified with a high power amplifier (HPA) and sent over the channel through the antenna. With an ideal channel, the receiver will be capable of inverting the process incurred at the transmitter, therefore, estimating the corresponding information bits without errors.

It only make sense to define the PAPR at the input of the HPA, i.e., for the time-domain OFDM signal $x(t)$: it is the ratio between the maximum instantaneous power of the signal and its average power. However, in this paper, as in many solutions found in the literature, the PAPR reduction algorithm is accomplished in the discrete domain as shown in Figure 1. Therefore, we define the PAPR as

$$\text{PAPR}\{\mathbf{x}_L\} = \frac{\max\limits_{n} |x_L[n]|^2}{\text{E}\{|x_L[n]^2|\}}, \qquad \forall n\,, \tag{2}$$

where $\text{E}\{\cdot\}$ denotes the expectation operation. Note that in (2) the PAPR is defined for an interpolated version of the original OFDM signal. The interpolation factor is indicated by L. The interpolation process, which can be accomplished in frequency or in time-domain, is necessary to approximate the continuous time-domain signal, i.e., the larger L, the more $x_L[n]$ resembles $x(t)$. Without this oversampling, the maximum peak of $x(t)$ will not be sampled and therefore the PAPR could be much lower than for $x(t)$. In the literature it is found that choosing $L \geq 4$ could be enough for a good trade-off between an approximation of $x(t)$ and computing complexity. In the rest of the paper it is understood that the signal processing is accomplished in already interpolated signals, therefore there is no need of indicating the interpolation factor L.

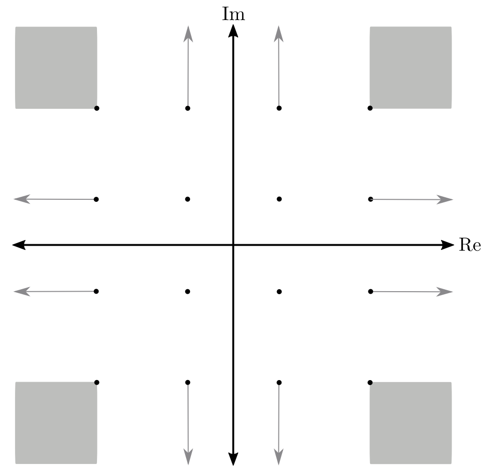

Fig. 2: The complex plane depicting a 16-QAM constellation and its corresponding ACE outer-point extensions

III. Active Constellation Extension

In this section, a short description of the active constellation extension (ACE) algorithm is presented while a detailed description can be found in [6].

The idea behind ACE algorithm is straightforward and given in Figure 2. This figure depicts the complex plane. The black dots are the elements of $\mathbb{M}$, which in this case correspond to the elements of a 16-QAM modulation scheme. ACE uses a non-bijective function to change the constellation in such a way that more flexibility is added to the outer points of the constellation. The outer points are allowed to extend outward in one or two directions under the restriction that the minimum euclidean distances determined by the original modulation scheme is not reduced. With this relaxation, ACE searches values for the outer points in order to reduce the PAPR. Thus, in Figure 2, the gray areas represent the corner-point extension regions while the gray lines represent the extension paths for the side point. From here it is clear, that the interior points can not be remapped.

The search for the appropriate coordinates of the outer points within the allowed boundaries of the modified constellation, which will be able to reduce the PAPR to a minimum, is translated to a special case of a quadratically-constrained quadratic program (QCQP). Obtaining the optimal solution can be very difficult. For this reason, three practical algorithms for ACE implementation are given in [6]. These are good suboptimal solutions of the previously mentioned ACE optimization problem. The first algorithm is the projection onto convex sets (POCS), which has optimality properties but converges very slowly. The other two algorithms are the approximate gradient-project (AGP) and the smart gradient-project (SGM) method. The last two methods converge faster than POCS toward a low-PAPR solution and are described briefly in this section.

A. Approximate Gradient-Project Method

The approximate gradient-project (AGP) method is an iterative algorithm that searches for a PAPR reduction by minimizing a peak below some amplitude A. Its goal is to reduce the peak to a minimum. The AGP algorithm is defined as follows:

1) Given a frequency-domain OFDM symbol vector $\mathbf{X}$, determine and store the allowable extension directions for each subcarrier. Transform it to the time-domain as $\mathbf{x}^i = \text{IFFT}_N\{\mathbf{X}\}$, with i denoting the iteration index. Set $i = 0$.
2) Clip the time domain signal for all n as follows:
$$x^i_{\text{clip}}[n] = \begin{cases} x^i[n], & \text{if } |x^i[n]| \leq A \\ Ae^{j\theta[n]}, & \text{if } |x^i[n]| > A \end{cases}, \tag{3}$$
where $\theta[n] = \measuredangle x^i[n]$.
3) Compute only the corresponding clipped signal portion, i.e.,
$$c^i_{\text{clip}}[n] = x^i_{\text{clip}}[n] - x^i[n] \,. \tag{4}$$
4) Transform the clipped signal portion to its frequency domain,
$$\mathbf{C}^i_{\text{clip}} = \text{FFT}_N\{\mathbf{c}^i_{\text{clip}}\} \,. \tag{5}$$
Please note that due to the linearity property of DFT, with (5) it can be easily computed
$$X^i_{\text{clip}}[k] = X[k] + C^i_{\text{clip}}[k] \,, \tag{6}$$
for every k, where $X[k] \in \mathbb{M}$ is an already known signal form Step 1. Computing (5) is less expensive than transforming $\mathbf{X}^i_{\text{clip}} = \text{FFT}_N\{\mathbf{x}^i_{\text{clip}}\}$ directly because most of the symbols of $c^i_{\text{clip}}[n]$ are expected to be 0.
5) Compute $\bar{\mathbf{C}}^i_{\text{clip}} = \zeta\{\mathbf{C}^i_{\text{clip}}\}$. We define in this paper a function $\zeta\{\cdot\}$ that gives only the components of $\mathbf{C}^i_{\text{clip}}$ which are acceptable extensions directions and set all remaining directions to zero. Compute the time domain signal $\bar{\mathbf{c}}^i = \text{IFFT}_N\{\bar{\mathbf{C}}^i_{\text{clip}}\}$. Please note that this step implies that once a vector is extended to some point in any iteration, it can not be reversed in subsequent iterations.
6) Finally, determine the new, time-domain OFDM symbol by computing
$$x^{i+1}[n] = x^i[n] + \mu\, \bar{c}^i[n] \tag{7}$$
for all n, where μ is some gradient step size. Its value can be chosen after experimental optimization or by the smart gradient-project (SGP) method introduced in the next section.
7) If PAPR has reached some lower threshold or a maximum iteration count has been reached, finish the algorithm, otherwise, update $i = i + 1$ and return to Step 2.

It turns out that the function $\zeta\{\cdot\}$ does not allows the reverse of any vector, which is a suboptimal approach but an efficient solution if $\bar{\mathbf{c}}^i$ must be computed before choosing μ.

B. Smart Gradient-Project Method

As pointed out in Step 6 of the AGP method, the parameter μ must still be chosen according to some criteria. This parameter controls the convergence speed of the algorithm, which is guaranteed to converge to a minimum for a very small μ. Nevertheless, a small step size of μ may lead to a slower convergence.

To find the optimal value of μ, a quadratic formula must be solved fore every iteration. This could not be feasible for real systems. Therefore, rather than finding the optimal values for μ, in [6] a suboptimal search is proposed, i.e., the smart gradient-project (SGP) method. The algorithm is as follows:

1) Given $\mathbf{x}^i = x^i[n]$ for $n = 0, 1, ..., N-1$, compute
$$E = \max_n |x^i[n]| \tag{8}$$
and
$$n_{max} = \underset{n}{\text{argmax}}\, |x^i[n]| \,. \tag{9}$$
2) For all n, compute the projection of $\bar{c}^i[n]$ along the phase angle of $x^i[n]$ as
$$c_{\text{proj}}[n] = \frac{Re\{x^i[n]\ \bar{c}^*[n]\}}{|x^i[n]|} \,, \tag{10}$$
where $(\cdot)^*$ denotes the complex conjugate.
3) For all n for which $c_{\text{proj}}[n] > 0$, compute $\mu[n]$ as follows:
$$\mu[n] = \frac{E - |x^i[n]|}{c_{\text{proj}}[n] - c_{\text{proj}}[n_{max}]} \,. \tag{11}$$
4) From (11) select the minimum value, i.e.,
$$\mu_{min} = \min_n \mu[n] \,, \tag{12}$$

and if $\mu_{min} > 0$ then continue with (7) with $\mu = \mu_{min}$, otherwise stop the PAPR reduction algorithm.

Note that $\mu[n]$ given in (11) must be computed for all n in every iteration of the AGP method.

IV. Modified ACE Algorithm

The modified ACE (mACE) algorithm proposed in this paper is of a similar complexity in comparison to the AGP method introduced in Section III-A and it performs similar to the SGP presented in Section III-B in terms of its PAPR reduction capability. The main idea is to avoid the computation of $\mu[n]$ for all n as in SGP, i.e., avoiding thus the necessity of computing (8)-(12) in every iteration.

The proposed mACE algorithm requires just a little modification in Steps 5 and 6 of AGP, nevertheless, for the sake of completeness we detail the complete steps of the algorithm here.

1) Given a frequency-domain OFDM symbol vector $\mathbf{X}^i$, determine and store the allowable extension directions for each subcarrier. Transform it to the time domain as $\mathbf{x}^i = \text{IFFT}_N\{\mathbf{X}^i\}$, with i denoting the iteration index. Set $i = 0$.
2) Clip the time domain signal for all n as follows:
$$x^i_{\text{clip}}[n] = \begin{cases} x^i[n], & \text{if } |x^i[n]| \le A \\ Ae^{j\theta[n]}, & \text{if } |x^i[n]| > A \end{cases}, \tag{13}$$
Where A is an arbitrary amplitude as defined for SGP method, and where $\theta[n] = \measuredangle x^i[n]$.
3) Compute the corresponding clipped signal portion, i.e.,
$$c^i_{\text{clip}}[n] = x^i_{\text{clip}}[n] - x^i[n]\,. \tag{14}$$
4) Transform the clipped signal portion to its frequency domain,
$$\mathbf{C}^i_{\text{clip}} = \text{FFT}_N\{\mathbf{c}^i_{\text{clip}}\}\,. \tag{15}$$
5) For simplicity, we define $X^{-1}_{\text{fc}}[k] = X^0[k]$ just for $i = 0$ and compute the clipped signal as:
$$\hat{X}^i_{\text{clip}}[k] = X^{i-1}_{\text{fc}}[k] + C^i_{\text{clip}}[k]\,, \tag{16}$$
where $X^{i-1}_{\text{fc}}[k]$ is defined in the next step.
6) Apply the frequency-domain constraints, e.g., as stated in Figure 2 for a 16-QAM modulation scheme,
$$X^i_{\zeta}[k] = \zeta\{\hat{X}^i_{\text{clip}}[k]\}, \quad \forall k\,, \tag{17}$$
where $\zeta\{\cdot\}$ is the function that projects the vector of $\hat{\mathbf{X}}^i_{\text{clip}}$ to the allowable extensions areas. Furthermore, weight a vector that falls inside the allowable extension area or path and compute $X^i_{\text{fc}}[k]$ as follows:
$$X^i_{\text{fc}}[k] = (X^i_{\zeta}[k] - X^0[k]) * \beta + X^0[k], \tag{18}$$
and from (18) proceed to compute $\bar{\mathbf{C}}^i_{\text{clip}}$ as
$$\bar{C}^i_{\text{clip}}[k] = X^i_{\text{fc}}[k] - X^{i-1}_{\text{fc}}[k]\,. \tag{19}$$
Compute the time domain signal $\bar{\mathbf{c}}^i = \text{IFFT}_N\{\bar{\mathbf{C}}^i_{\text{clip}}\}$.
7) Finally, determine the new time domain OFDM symbol by computing
$$x^{i+1}[n] = x^i[n] + \mu\,\bar{c}^i[n] \tag{20}$$
for all n. The value of μ is constant over all iteration and for all n. It can be chosen after experimental optimization.
8) If PAPR has reached some lower threshold or a maximum iteration count, finish the algorithm, otherwise, update $i = i + 1$ and return to Step 2.

Note that the second term on the right-hand side of (7) and (20) can be also performed in the frequency-domain. In AGP this is equivalent to $\mu\,\bar{c}^i[n] = \text{IFFT}_N\{(\zeta\{\mathbf{X}^i_{\text{clip}}\} - \mathbf{X})\,\mu\}$ and in mACE this will be performed in (19) as $\mu\,\bar{c}^i[n] = \text{IFFT}_N\{(\mathbf{X}^i_{\text{fc}} - \mathbf{X}^{i-1}_{\text{fc}})\,\mu\}$.

AGP as well as mACE compensate the frequency-domain constraints of $\zeta\{\cdot\}$ with an increment of power in the shifted symbols. AGP does this with μ in (7), however, mACE achieves this in two stages with μ and β. Besides the introduction of β, mACE utilizes both original signals and signals computed in the previous iteration. For instance, the computation of $\bar{\mathbf{C}}^i_{\text{clip}}$ in (20) includes a result from a previous iteration for mACE while for AGP it can be tracked back to (4) which requires results from the same iteration for its computation. Thus, mACE aggregates some extra degrees of freedom for its solution, which gives some advantages in comparison to AGP and SGP.

One of the advantages of mACE in comparison to AGP and SGP is that it permits reverse of extended vectors projected on the allowable extension in any previous iterations. This adds extra flexibility towards an optimal solution. Another advantage, and the main reason of the development of mACE, is that it can save all computation of SGP. The parameters μ and β are constants that must be chosen properly, they can be computed off-line by means of simulation. As it is shown in the next section, with properly chosen μ and β, mACE may even perform better than SGP method.

V. Simulation Results and Discussion

In this section the performance of mACE is evaluated. The performance of SGP is also reproduced and used as a benchmark.

A. Parameter Settings

The proposed PAPR reduction algorithm is evaluated using 10^7 random generated OFDM symbols with $N = 256$ subcarriers. We choose an oversampling factor of $L = 4$. The clipping magnitude is set to $A = 4.86$ dB above the average power of the OFDM symbol and the PAPR threshold is set to 6 dB. We use theses values in order to reproduce the results of SGP presented in [6] and use them as benchmark. The algorithms are tested for two modulation schemes: QPSK

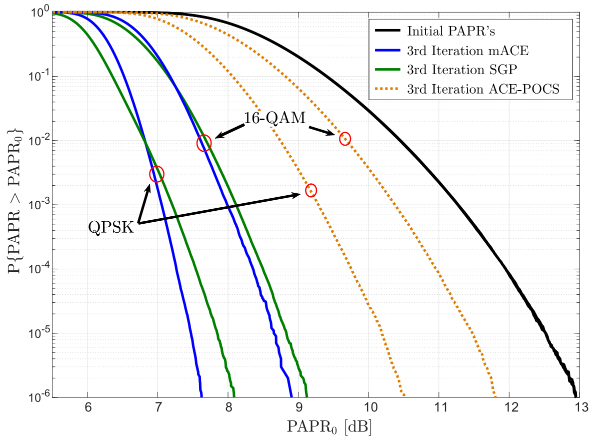

Fig. 3: Performance comparison in terms of ccdf of the PAPR reduction by mACE and SGP algorithms for 10^7 OFDM symbols with $N = 255$ subcarriers, for QPSK and 16-QAM. The interpolation factor is $L = 4$.

and 16-QAM and the maximum number of iterations is set to $i_{max} = 3$.

B. *Simulation Results*

The N sample values of an OFDM symbol $x[n]$, $n = 0, ..., N-1$, in time domain can be considered as a zero-mean unit-variance complex Gaussian random variable. The PAPR is determined by its largest peak among all samples, therefore, a common performance measure in the literature is the complementary cumulative distribution function (ccdf) of the PAPR, i.e., $\mathrm{P}\{PAPR > PAPR_0\}$, the probability that the PAPR of an OFDM symbol exceeds a give threshold $PAPR_0$, with $\mathrm{P}\{\cdot\}$ denoting probability.

The ccdf of the PAPR reduction algorithms are depicted in Figure 3 for QPSK and 16-QAM modulation schemes. It can be noticed that mACE performs very similar to SGP, and even it achieves better results than SGP for lower $\mathrm{P}\{PAPR > PAPR_0\}$. The extra PAPR reduction gains achieved by mACE for QPSK and 16-QAM modulation schemes are up to 0.5 dB and 0.2 dB respectively for a $\mathrm{P}\{PAPR > PAPR_0\}$ of 10^{-6} with less computation. This performance is due to the fact that mACE allows reverse extension vectors while SGP does not, which gives extra flexibility towards an optimal solution. In Section III-B, the computational burden of SGP can be noticed. In each iteration, SGP needs to compute the equivalent to an FFT in order to determine $\mu[n]$ for all n. This is equivalent to the savings of mACE in comparison to SGP. On the other hand, mACE has to multiply β with the signal in (18), this small increment in complexity wrt. AGP can be well justified by its performance. The results in Figure 3 were obtained for the system with QPSK by choosing $\mu = 2.5$ and $\beta = 1.5$, and for the system with 16-QAM by setting $\mu = 4.5$ and $\beta = 1.65$. These values were chosen for comparison purposes, they can further be optimized by simulation.

VI. Conclusions

In this paper, we introduce and evaluate the mACE algorithm for the PAPR reduction in OFDM systems. Its performance has been measured and compared with the SGP method which, compared to other ACE approaches, so far has reached the best PAPR reduction after a couple of iterations. By choosing suitable values for μ and β, it is shown that mACE performs better than the SGP method and it requires less computation. For instance, in an OFDM system with QPSK modulation scheme, mACE outperforms the SGP for aproximately 0.5 dB at a $\mathrm{P}\{PAPR > PAPR_0\}$ of 10^{-6} with the constant pairs ($\mu = 2.5$, $\beta = 1.5$). In an OFDM system with 16-QAM modulation scheme, mACE performs just 0.2 dB better than SGP at a ccdf of 10^{-6} with the constant pairs ($\mu = 4.5$, $\beta = 1.65$). For symbols with $N = 255$ subcarriers, the mathematical operations saved by mACE can be compared to computing an FFT in each iteration since μ and β are constants searched off-line by simulations. To this end, mACE realizes an appropriate trade-off between PAPR reduction and computational complexity, which makes it a viable option for implementations in real time systems.

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