ANALYSIS OF DIFFERENT DISPLACEMENT ESTIMATION ALGORITHMS FOR DIGITAL TELEVISION SIGNALS

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ABSTRACT

In this contribution five algorithms for two-dimensional displacement estimation in digital television scenes based on the differential method are mathematically analyzed. The estimation criterion and the mathematical formulation for each algorithm are derived. The estimation procedures and the simplifications are explained. Based on the analytic results a new estimation algorithm is derived that estimates the cross correlation peak of translatory displaced luminance signals within small image segments. This analysis shows the advantage of the new algorithm with respect to the convergency.

1. INTRODUCTION

Several algorithms which estimate the displacement of moving objects between two successive television frames have already been published. With respect to the mathematical formulation these algorithms are divided into three classes /1/:

- Fourier methods
- correlation methods
- methods of differentials.

The third class of algorithms is particularly important for real-time digital television signal processing, because, in many cases, signal transformations or matching procedures are too complex. Concerning calculation complexity the algorithms based on differential methods

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can be subdivided into three groups of algorithms, where

1. One displacement estimate is calculated for the entire moving area /2/.
2. The displacement estimates are calculated within the moving area for small rectangular image segments /3,4/.
3. The displacement estimates are calculated recursively pel by pel /5,7/.

When weighting the advantages and drawbacks of each, it is of interest to point out the differences between the algorithms with respect to the formulation and the simplifying assumptions. In this contribution four displacement estimation algorithms are compared on the base of a mathematical analysis. In addition a new algorithm is presented and compared to the others by some experimental results.

2. DISPLACEMENT ESTIMATION BY THE METHOD OF DIFFERENTIALS

2.1 NONRECURSIVE DISPLACEMENT ESTIMATION ALGORITHM OF LIMB AND MURPHY

Limb and Murphy /2/ have proposed an algorithm that calculates one displacement estimate for the entire moving area. The algorithm is based on the two assumptions that the television scene contains one moving object and that the movement can be described by pure horizontal translation. The moving area is determined by a simple frame difference thresholding algorithm. By a heuristic approach the sum over the frame differences within the moving area is found to be a function of the displacement. However, this function depends upon the size of the moving area and the amount of detail within. In order to eliminate these influences the sum over the frame differences is divided by the sum over the spatial luminance differences of adjacent pels, i.e. the element differences. Using this normalization, the amount $|\mu|$ of the horizontal displacement estimate is determined by

$$|\mu| = \frac{\sum\limits_{M} |\Delta_t I(m,n)|}{\sum\limits_{M} |\Delta_m I(m,n)|} \qquad (1)$$

$\Delta_t I$: frame difference
$\Delta_m I$: element difference
M : moving area

In order to determine the direction of the displacement, each sum in (1) is divided into two parts in which the summation is carried out with respect to the sign of the element and frame differences. The difference of both parts specifies the estimate of the horizontal displacement

$$\mu = \frac{\sum\limits_{M} |\Delta_t I(m,n)| \cdot sb(\Delta_t I) \not\equiv sb(\Delta_m I) - \sum |\Delta_t I(m,n)| \cdot sb(\Delta_t I) \equiv sb(\Delta_m I)}{\sum\limits_{M} |\Delta_m I(m,n)| \cdot sb(\Delta_t I) \not\equiv sb(\Delta_m I) - \sum |\Delta_m I(m,n)| \cdot sb(\Delta_t I) \equiv sb(\Delta_m I)} \qquad (2)$$

$$sb(\cdot) = \begin{matrix} 0 \text{ , if } (\cdot) \text{ is negative} \\ 1 \text{ , if } (\cdot) \text{ is positive} \end{matrix}$$

$\equiv$ LOGICAL EQUIVALENCE
$\not\equiv$ LOGICAL ANTIVALENCE

which has the advantage of a relatively simple hardware realization. An experimental evaluation by the authors results in a good performance for displacements of up to 2.5 pel per frame.

In Figure 1 an example for displacement estimation using this algorithm is demonstrated.

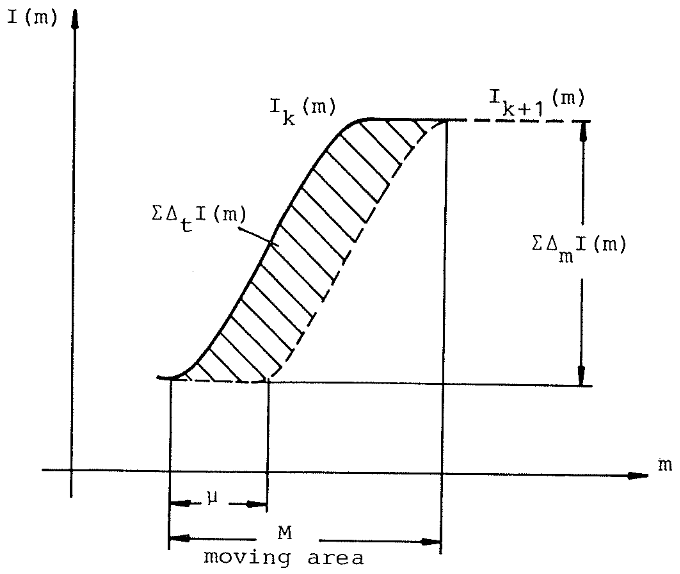


Figure 1. One-dimensional example for displacement estimation using equation (2)

The line denoted by I(m) represents the one-dimensional luminance function of a moving object, the dashed line shows the function in the succeeding frame. Using the displacement estimation algorithm (2) the frame differences are summed over the hatched part in Figure 1 and represent approximately a measure of the area of a parallelogram. Dividing this value by the height of the parallelogram, i.e. the sum over the element differences along m, we get the displacement $\hat{\mu}$.

The problem of displacement estimation is led back to the calculation of the base of a parallelogram knowing its area and height. This implies the requirement that the integral over the frame differences is a linear function in the displacement μ. In this case equation (2) gives an exact estimate for μ. However, in most cases Figure 1 does not hold for real television signals. To take into account the statistical properties of television signals the expected values should be used for an analytic evaluation. Before going into further detail the algorithm of Cafforio and Rocca is derived because it is similar to that of Limb and Murphy.

2.2 NONRECURSIVE DISPLACEMENT ESTIMATION ALGORITHM OF CAFFORIO AND ROCCA

A displacement estimation algorithm for small image segments has been proposed by Cafforio and Rocca /3/. Considering a moving object in a television scene, the two-dimensional luminance signal of two succeeding frames is denoted by

$$I_k(m,n) = I_{k-1}(m-\mu,n-\nu) \tag{3}$$

μ,ν : horizontal and vertical displacement

if the movement consists of pure translation. Calculating the frame differences we get

$$\Delta_t I(m,n) = I_k(m,n) - I_{k-1}(m,n) \tag{4}$$

and approximating (4) by the truncated Taylor series expansions we get

$$\begin{aligned}\Delta_t I(m,n) &= \mu\cdot\frac{\partial}{\partial_m} I_{k-1}(m,n) + \nu\cdot\frac{\partial}{\partial_n} I_{k-1}(m,n) + \varepsilon(m,n)\\ &\approx \mu\cdot\Delta_m I_{k-1}(m,n) + \nu\cdot\Delta_n I_{k-1}(m,n) + \varepsilon(m,n)\end{aligned} \tag{5}$$

where the partial derivatives are approximated by the spatial differences. The expression $\varepsilon(m,n)$ accounts for the error due to the truncation of the Taylor series expansion of $I_{k-1}(m,n)$ around the location (m,n). Assuming that the error term is not correlated with the signal and has an even probability density function, the optimal estimates for μ and ν are obtained by linear regression. For example the horizontal estimate $\hat{\mu}$ is

$$\hat{\mu} = \frac{\sigma_{\Delta_t I}}{\sigma_{\Delta_m I}} \cdot \frac{\rho_{\Delta_t I \Delta_m I} - \rho_{\Delta_m I \Delta_n I}\cdot\rho_{\Delta_t I \Delta_n I}}{1 - \rho^2_{\Delta_m I \Delta_n I}} \tag{6}$$

with the variance σ^2 and the correlation coefficient ρ

$$\sigma^2_{\Delta_m I} = \frac{1}{K}\sum_{i,j} \Delta_m I^2(m_i,n_j) \tag{7}$$

$$\rho_{\Delta_t I \Delta_m I} \cdot \sigma_{\Delta_t I} \cdot \sigma_{\Delta_m I} = \frac{1}{K} \sum_{i,j} \Delta_t I(m_i, n_j) \cdot \Delta_m I(m_i, n_j) \tag{8}$$

The summation is carried out over all the picture element locations (m_i, n_j) within the image segment. If the correlation between $\Delta_m I$ and $\Delta_n I$ is assumed to be negligible and the approximation

$$\rho_{\Delta_t I \Delta_m I} \cdot \sigma_{\Delta_t I} = \frac{1}{K} \sum_{i,j} \Delta_t I(m_i, n_j) \cdot \mathrm{sign}\ \Delta_m I(m_i, n_j) \tag{9}$$

is valid, the estimate is simplified to

$$\hat{\mu} = \frac{\sum_{i,j} \Delta_t I(m_i, n_j) \cdot \mathrm{sign}\ \Delta_m I(m_i, n_j)}{\sum_{i,j} |\Delta_m I(m_i, n_j)|} \tag{10}$$

which is similar to equation (2). Writing equation (1o) without using the approximation (9) we get

$$\hat{\mu} = \rho_{\Delta_t I \Delta_m I} \cdot \frac{\sigma_{\Delta_t I}}{\sigma_{\Delta_m I}} \tag{11}$$

$$= \frac{1}{N} E\left[\{I_{k-1}(m,n) - I_{k-1}(m+1,n)\} \cdot \{I_k(m,n) - I_{k-1}(m,n)\}\right] \tag{12}$$

$$= \frac{1}{N} \Big[E\left[I_{k-1}(m+1,n) \cdot I_{k-1}(m,n)\right] - E\left[I^2_{k-1}(m,n)\right]$$

$$+ E\left[I_{k-1}(m,n) \cdot I_k(m,n)\right] - E\left[I_{k-1}(m+1,n) \cdot I_k(m,n)\right] \Big] \tag{13}$$

with

$$N = E\left[\{I_{k-1}(m,n) - I_{k-1}(m+1,n)\}^2\right] \tag{14}$$

The first two expressions in equation (13) are independent of the displacement and represent an approximation of the first derivative of the autocorrelation function around the peak. A model of the correlation function based on the measurements of real-world television signals is used for further evaluation of equation (13). Figure 2 shows the two-dimensional correlation function of a monochrome television signal. If pure translation is assumed Figure 3 shows the relation between the actual displacement and the estimate according to equation (13) . For simplicity in this case only the horizontal displacment μ and its estimate $\hat{\mu}$ have been evaluated.

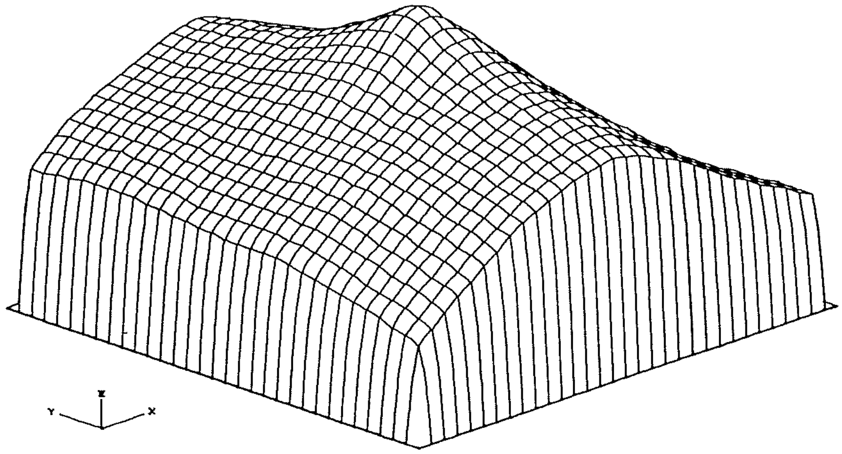


Figure 2. Cross correlation function of two-dimensional luminance signals

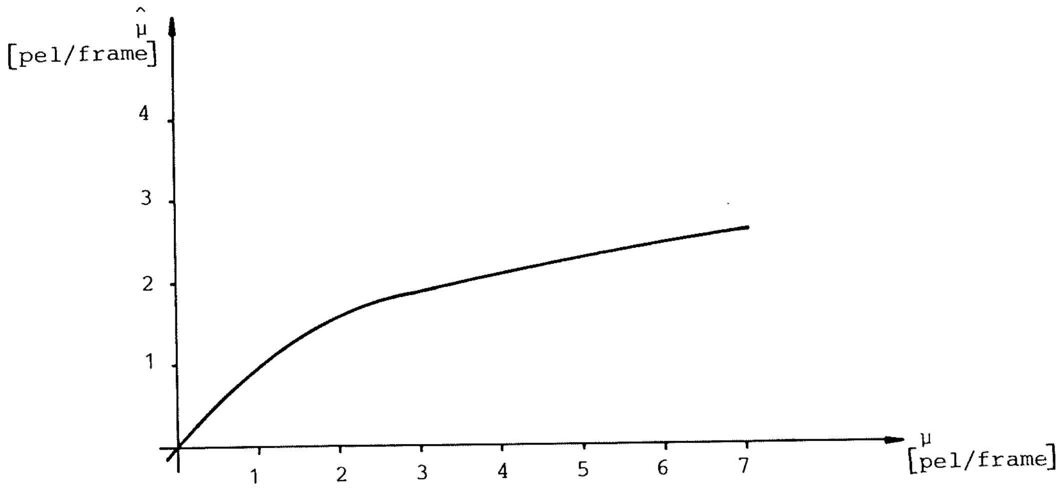


Figure 3. Relation between the actual displacement μ and the estimate $\hat{\mu}$ according to equation (13)

Figure 3 shows a linear relation between μ and $\hat{\mu}$ up to displacements of about 1.5 pel per frame. Exceeding this value equation (13) obviously fails. This may be the reason why the authors of /3/ have evaluated a good performance up to 2.5 pel per frame. For estimating larger displacements they propose spatial low pass filtering of the luminance signals. This would result in a deformation of the correlation function slope in Figure 2, so that the linear range of the function in Figure 3 is enlarged.

Another displacement estimation algorithm which is essentially identical to Rocca's has been proposed by Schalkoff and McVey /4/ and evaluated by Dinse, Enkelmann and Nagel /8/.

One component of the estimate is determined by

$$\hat{\mu} = \frac{1}{\sum_{i,j} \Delta_m I^2 \cdot \sum_{i,j} \Delta_n I^2 - 2 \cdot \sum_{i,j} \Delta_m I \Delta_n I} \cdot$$

$$\left(\sum_{i,j} \Delta_n I^2 \cdot \sum_{i,j} \Delta_m I \Delta_t I - \sum_{i,j} \Delta_m I \Delta_n I \cdot \sum_{i,j} \Delta_n I \Delta_t I \right) \qquad (15)$$

which is identical to equation (6).

This algorithm has been implemented and evaluated in /8/. The measurements have shown that the sums over the mixed derivatives in (15) are between one and two orders of magnitude less than the principal diagonal elements and can be neglected in the calculation of estimates. The reason is that these expressions represent the correlation between the partial derivatives in the m- and n-directions which can be assumed to tend to zero. In this case equation (15) becomes

$$\hat{\mu} = \frac{\sum_{i,j} \Delta_m I \Delta_t I}{\sum_{i,j} \Delta_m I^2} \qquad (16)$$

$$= \rho_{\Delta_t I \Delta_m I} \cdot \frac{\sigma_{\Delta_t I}}{\sigma_{\Delta_m I}} \qquad (17)$$

which is identical to equation (11).

2.3 RECURSIVE DISPLACEMENT ESTIMATION ALGORITHM OF NETRAVALI AND ROBBINS

An elegant method for recursive displacement estimation has been published by Netravali and Robbins /5/. In order to use a pel recursive gradient technique the quadratic error function is given by the squared displaced frame difference DFD^2 which is to be minimized,

$$DFD^2_{m,n} \rightarrow \min \tag{18}$$

with

$$DFD_{m,n} = I_k(m,n) - I_{k-1}(m-\mu_{i-1}, n-\nu_{i-1}) \tag{19}$$

and the displacement estimates from the previous pel using vectorial representation

$$\vec{D}_{i-1} = (\mu_{i-1}, \nu_{i-1}) \quad . \tag{2o}$$

This formula leads to the recursion formula

$$\hat{\vec{D}}_i = \vec{D}_{i-1} - \varepsilon \cdot DFD \cdot \nabla I_{k-1}(m-\mu_{i-1}, n-\nu_{i-1}) \tag{21}$$

∇ : NABLA Operator

where ∇I_{k-1} approximates the spatial gradient of the displaced luminance in frame k-1 by differences.

In the following equation the statistic expectancies of DFD and I_{k-1} are used instead of just one value, because luminance signals generally represent non-deterministic source signals. If we calculate the update term of (21) with an initial estimate $\mu_{i-1} = \nu_{i-1} = 0$, we get for instance the horizontal displacement estimate

$$\hat{\mu} = -\varepsilon \cdot E\left[DFD_{m,n} \cdot \Delta_m I_{k-1}(m,n)\right]$$

$$= -\varepsilon \cdot E\left[\{I_k(m,n) - I_{k-1}(m,n)\} \cdot \{I_{k-1}(m,n) - I_{k-1}(m+1,n)\}\right] \tag{22}$$

which is identical to equation (12), if we choose $\varepsilon = \frac{1}{N}$. The negative sign is due to the definition of DFD in (19). In contrast to /3/ the recursive procedure tries to keep the estimate within the linear range of the curve in Figure 3.

2.4 NEW DISPLACEMENT ESTIMATION ALGORITHMS

Let us consider again the estimation criterion which demands for minimizing the displaced frame differences

$$E[DFD^2_{m,n}] \rightarrow \min$$

$$\begin{aligned} E[DFD^2_{m,n}] &= E[\{I_k(m,n) - I_{k-1}(m-\mu_{i-1},n-\nu_{i-1})\}^2] \\ &= E[I^2_k(m,n)] + E[I^2_{k-1}(m-\mu_{i-1},n-\nu_{i-1})] \\ &\quad - 2\,E[I_{k-1}(m-\mu_{i-1},n-\nu_{i-1}) \cdot I_k(m,n)] \\ &= R_{I^2_{k-1}}(0) + R_{I^2_k}(0) \\ &\quad - 2\,R_{I_{k-1}I_k}(\mu-\mu_{i-1},\nu-\nu_{i-1}) \end{aligned} \tag{23}$$

R : correlation function

Using the gradient technique for recursive minimization we get the recursion formula for the displacement vector $\vec{D}$

$$\begin{aligned} \hat{\vec{D}}_i(\mu,\nu) &= \vec{D}_{i-1}(\mu,\nu) - \frac{\varepsilon}{2}\cdot\nabla E[DFD^2_{m,n}] \\ &= \vec{D}_{i-1}(\mu,\nu) + \varepsilon\cdot\nabla R_{I_{k-1}I_k}(\mu-\mu_{i-1},\nu-\nu_{i-1}) \end{aligned} \tag{24}$$

and, for instance, for the horizontal component

$$\hat{\mu}_i = \mu_{i-1} + \varepsilon \left. \frac{\partial R_{I_{k-1}I_k}}{\partial \mu} \right|_{\mu_{i-1}} \tag{25}$$

If we exchange the summation and the derivation we get for the update term in equation (25)

$$\frac{\partial R_{I_{k-1}I_k}}{\partial \mu} = \frac{\partial}{\partial \mu} \frac{1}{K} \sum I_{k-1}(m-\mu, n-\nu) \cdot I_k(m,n)$$

$$= - \frac{1}{K} \sum \frac{\partial}{\partial (m-\mu)} I_{k-1}(m-\mu, n-\nu) \cdot I_k(m,n)$$

$$= - E[I'_{k-1}(m-\mu, n-\nu) \cdot I_k(m,n)]$$

$$= - R_{I'_{k-1}I_k} \qquad (26)$$

An approximation of the first derivative of the luminance signal by spatial differences leads to the formula

$$R_{I'_{k-1}I_k} = E[\{I_{k-1}(m,n) - I_{k-1}(m+1,n)\} \cdot \{I_k(m,n)\}] \quad . \qquad (27)$$

For an initial value of the displacement $\mu_{i-1} = \nu_{i-1} = 0$ we get

$$\hat{\mu}_i = - \varepsilon [E[I_{k-1}(m,n) \cdot I_k(m,n)] - E[I_{k-1}(m+1,n) \cdot I_k(m,n)]] \qquad (28)$$

While equation (28) contains only a displacement dependent expression which can be interpreted as an approximation of the first derivative of the cross correlation function $R(\mu,\nu)$ in direction of μ the formulation according to equation (22) additionally contains two expressions which are independent from the displacement

$$- E[I^2_{k-1}(m,n)] + E[I_{k-1}(m,n) \cdot I_{k-1}(m+1,n)]$$

$$= - R_{I^2_{k-1}}(o) + R_{I^2_{k-1}}(1) \qquad (29)$$

The algorithm according to equation (24) requires the calculation of the gradient of the cross correlation function within a small image segment. If a minimum window size of only one pel is used, the displacement calculation would require one subtraction and multiplication, while equation (21) requires one subtraction additionally. Thus the gradient technique (24) has the advantage of a more simple hardware realization.

For a more detailed analysis we discuss the convergence behaviour of the gradient algorithm. If we have a symmetric cross correlation function the algorithm converges around the peak up to the two next locally adjacent minima. Assuming an actual displacement μ^* convergence is provided if the update term in equation (25) is smaller than the amount of $2|\mu_i - \mu^*|$,

$$2 \cdot |\mu_i - \mu^*| > \varepsilon \cdot \left| \frac{\partial}{\partial \mu} R_{I_{k-1} I_k} \right| . \qquad (30)$$

Equation (30) gives an upper limit for the constant ε in the neighbourhood of the actual displacement,

$$\varepsilon < 2 \cdot \lim_{\mu_i \to \mu^*} \left| \frac{\mu_i - \mu^*}{\frac{\partial}{\partial \mu} R_{I_{k-1} I_k}} \right| \qquad (31)$$

In order to determine the convergence rate we calculate the estimation error for successive recursions, i.e. the difference between estimate and actual displacement,

$$e(i) = \mu_i - \mu^* \qquad (32)$$

$$e(i+1) = \mu_{i+1} - \mu^* = \mu_i + \Delta\mu_i - \mu^* . \qquad (33)$$

In /9/ it has been shown that gradient methods are linear convergent. This means that the limit value of the ratio of successive estimation errors can be determined by

$$c = \lim_{i \to \infty} \frac{e(i+1)}{e(i)} \qquad (34)$$

By inserting (32) and (33) we get

$$c = \lim_{i \to \infty} \left(1 + \varepsilon \frac{\frac{\partial}{\partial \mu} R_{I_{k-1} I_k}}{\mu_i - \mu^*} \right) , \qquad (35)$$

and, because $\lim_{i\to\infty} \mu_i = \mu^*$, we obtain

$$c = 1 + \varepsilon \frac{\partial^2}{\partial\mu^2} R_{I_{k-1}I_k} \Big|_{\mu=\mu^*} \tag{36}$$

Equation (36) determines the convergence rate /9/ of the linear convergent recursion sequence (25) in dependence of ε. The smaller the amount of c the faster the algorithm converges to the optimum estimate μ^*. If we choose, for instance, an ε which is equal to the right side of relation (31) we will get a convergences rate

$$c = 1 + 2 \lim_{\mu\to\mu^*} \left| \frac{\mu_i - \mu^*}{\frac{\partial}{\partial\mu} R_{I_{k-1}I_k}} \right| \cdot \frac{\partial^2}{\partial\mu^2} R_{I_{k-1}I_k} \Big|_{\mu=\mu^*} \tag{36a}$$

Using the rule of l'Hospital we get

$$c = 1 + 2 \left| \left(\frac{\partial^2}{\partial\mu^2} R_{I_{k-1}I_k} \right)^{-1} \right|_{\mu=\mu^*} \cdot \frac{\partial^2}{\partial\mu^2} R_{I_{k-1}I_k} \Big|_{\mu=\mu^*} \tag{36b}$$

Because the second order derivative of the cross correlation function at the location $\mu = \mu^*$ is negative, equation (36b) becomes

$$c = -1. \tag{36c}$$

In this case the estimates oscillate between the initial estimates $+\mu_0$, $-\mu_0$, in the case $|c|>1$ divergence is provided. For every other chosen ε, whereby relation (31) is valid, equation (36) yields a convergence factor within the limits

$$-1 < c < 1 . \tag{36d}$$

For a graphical demonstration of the convergence rate in dependence of ε we approximate the estimation error as a function of the number of recursion steps using equation (33):

$$|c| = \frac{e(i+1)}{e(i)} \qquad i = 0,1,2,\ldots \quad .$$

This leads to an exponentially decreasing estimation error, for instance

$$e(i) = \alpha \cdot \exp\{\beta i\}. \tag{37}$$

Then we can determine β by

$$|c| = \exp\{\beta\} \tag{38}$$

and with a normalized estimation error $\alpha = 1$ we get

$$e(i) = \exp\{ \ln|c| \cdot i \} \tag{39}$$

where c is determined by equation (36) for a given ε.

In Figure 4a the estimation error with parameter $|c|$ is graphically shown. For a large number of recursion steps both curves $|c| = 0.5$ and $|c| = 0.25$ show sufficient small errors. Especially for only one or two steps the parameter curve $|c| = 0.25$ shows a better convergence concerning the remaining estimation error after truncation.

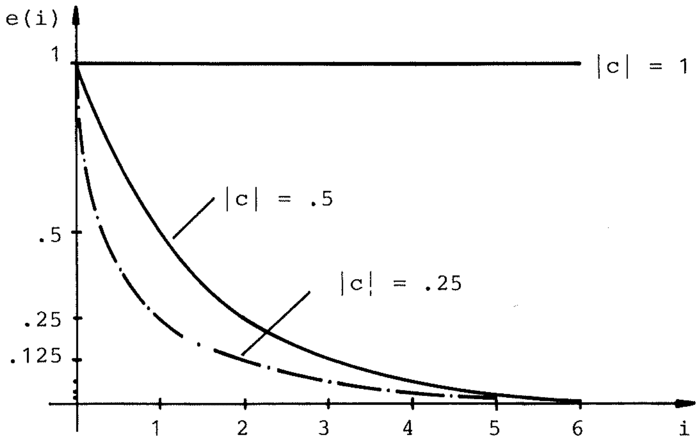


Figure 4a. Estimation error in dependence of the convergence factor $|c|$

The essential disadvantage of the gradient estimation technique is the constant stepwidth ε. By defining the stepwidth the convergency behaviour is determined only for the considered error function, in this case the squared displaced frame differences. Minimizing this function is traced back to maximize the cross correlation function where the direction of the recursive search is derived from its gradient. If the error function is slightly varying its shape the stability and the convergence of the algorithm are influenced. In order to avoid oscillations the stepwidth normally has to be chosen small enough. This would result in a large number of iterations for reaching the zero crossing. A faster convergence is obtained by adapting the stepwidth to the slope of the error function. If the second derivative of the cross correlation function is used for the determination of ε,

$$\varepsilon = \frac{1}{R''_{I_{k-1}I_k}} \tag{4o}$$

the algorithm is similar to the known Newton search method for one-dimensional signals. In Figure 4b an example is shown for the gradient technique and the Newton search technique.

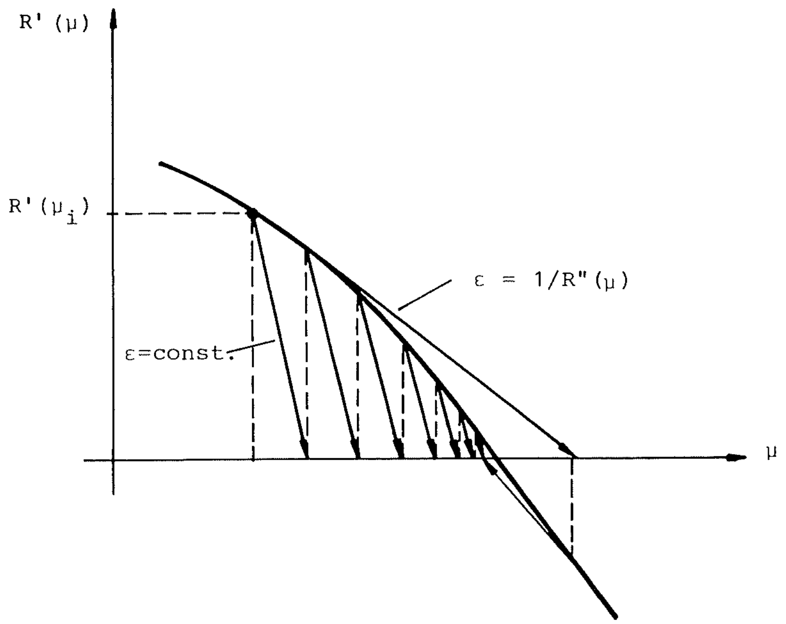


Figure 4b. Example of recursive zero crossing approximation using a constant and an adaptive step width

In /7/ a displacement estimation technique utilizing the derivatives of the cross correlation function for correlation peak estimation is proposed for two-dimensional monochrome television signals. The estimate for the horizontal displacement is given by

$$\hat{\mu}_i = \mu_{i-1} - \frac{\frac{\partial}{\partial \mu} R_{I_{k-1}I_k}}{\frac{\partial^2}{\partial \mu^2} R_{I_{k-1}I_k}}$$

$$= \mu_{i-1} + \frac{\sum_{i,j} \Delta_m I_{k-1} I_k}{\sum_{i,j} \Delta_m{}^2 I_{k-1} I_k} \tag{41}$$

where the derivatives are approximated by differences and calculated within small image segments. The recursion is carried out pel by pel, using the estimate from the previous pel as displacement prediction for the current pel. In general the Newton search technique has a quadratic convergence. If the cross correlation function is symmetrical, i.e. the derivative is S-shaped, it can be shown that the Newton algorithm has a third order convergence /9/. The convergence rate according to /6/ is then given by

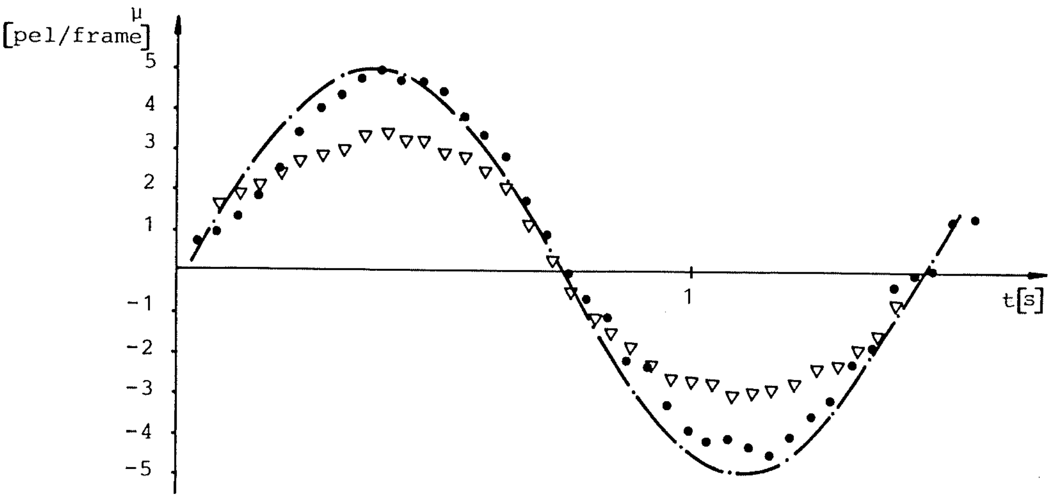
$$c = \lim_{i \to \infty} \frac{e(i+1)}{e(i)^3} \qquad (42a)$$

and in our case

$$c = \frac{1}{3} \frac{\left(\frac{\partial}{\partial \mu} R_{I_{k-1} I_k}\right)'''}{\left(\frac{\partial}{\partial \mu} R_{I_{k-1} I_k}\right)'} \Bigg|_{\mu = \mu^*} \qquad (42b)$$

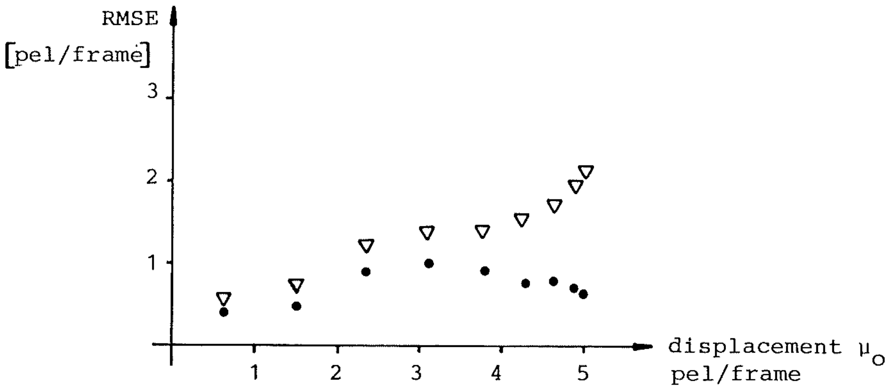
For the measured cross correlation function according to Figure 2 we get a convergence rate $c = -0.35$. This shows that the new algorithm converges relatively fast because the stepwidth is adapted to the slope of the cross correlation derivative.

In order to compare the performance of both algorithms /5/ and /7/ an experimental evaluation has been done by means of computer simulations. The input signal consists of a digitized monochrome television signal containing a swinging pendulum of 20 x 10 pel size. The known horizontal displacement can be descibed by a sinusoidal function and serves as reference function. Figure 5 shows the actual displacement as reference function and the estimate values. In Figure 6 the RMSE of all estimates within one field is shown. For displacements up to 2.5 pel per frame the RMSE of the gradient algorithm lies between 0.7 and 1.3 pel per frame. For displacements above 2.5 pel per frame the algorithm does not converge fast enough due to the small constant stepwidth ε. The RMSE of the Newton correlation peak estimation remains small and varies only slightly with the amount of the displacement.



∇ : gradient algorithm
• : correlation peak estimation algorithm
μ : horizontal displacement component
f : field number

Figure 5. Estimates and reference displacement for the horizontal displacement of the pendulum swing



∇ : gradient algorithm
• : correlation peak estimation algorithm

Figure 6. RMSE as a function of the reference displacement μ_o

3. CONCLUSION

The analyzation has shown that four of the discussed displacement algorithms basically utilize equation (11). For small displacements each algorithm has a good performance. For larger displacements the recursive gradient technique is expected to have a better performance. But as the stepwidth has to be chosen small in order to avoid oscillations the algorithm does not converge fast enough, i.e. it requires too many iterations for large displacements. This may be the reason for the relatively high root mean square error which has been measured on real television sequences. This analysis has shown that the calculation complexity of the gradient algorithm can be reduced. An improved convergence can be achieved by using the Newton search method for displacement estimation. In this case the stepwidth is derived from the quotient of the first and second order derivatives of the cross correlation function of the displaced luminance signals. A new displacement estimation algorithm which works on the base of the Newton search method has been proposed. The improvement with respect to the convergence could be proved by some experimental evaluations. However, in special cases the range of convergence of the new algorithm may be decreased. Therefore further investigations are provided for optimizing the stepwidth adaptation with respect to a fast convergence within an enlarged range of stability.

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