Three-Dimensional Reconstruction of Casting Defects from a Limited Number of X-ray Projections

Carsten Lehr
Institut für Theoretische Nachrichtentechnik und Informationsverarbeitung
University of Hannover

Appelstr. 9A
D-30167 Hannover
Germany

In the area of industrial quality inspection often only a very limited number of X-ray projections is available for the tomographic reconstruction. This is due to time- and cost reasons. Partly the aspect ratio of the components prevents a check from all sides. Conventional tomographic algorithms are not applicable in such cases. In this contribution a new approach for three-dimensional binary image reconstruction from a limited number of X-ray projections, applied to the reconstruction of casting defect, is presented. The reconstruction is carried out in a two step procedure. In a preprocessing step the reconstruction area is limited to the regions of interest around the defects. In these regions an iterative tomographic reconstruction procedure is employed. For the regularization of the tomographic reconstruction problem the maximum entropy principle is used in connection with a procedure for the binarisation of the reconstruction results. The reconstruction is carried out independently of restricting a priori assumptions over the shape of the defects.

1 Introduction

The further spreading of tomographic testing methods within the area of industrial quality inspection is limited at present by different factors of influence. Traditional tomographic systems are too complex and expensive for several applications. The necessary inspection time, using projections all around the sample, is much too long [1]. Frequently also the aspect ratio of the components prevents the radiographic testing from all sides necessary for the reconstruction. Therefore there is large interest in tomographic approaches, which operate on the basis of simple radiographs (reduction of the hardware expenditure) and which enable a reconstruction using only a few radiographs. In such cases, the tomographic reconstruction problem is strongly underdetermined and inconsistently due to different error influences (calibration error, noise), conventional tomographic algorithms (filtered backprojection, transformation methods) are not suitable. A priori information about the object under investigation has to be used together with a regularization approach in order to receive a unique solution.

The reconstruction of casting defects is a special application of the reconstruction of a default region (e.g. air) in an otherwise homogeneous object. Therefore the image reconstruction problem becomes the estimation of binary objects. In the literature different approaches are suggested. Several approaches are based on a description of the objects under investigation by parameterized functions, e.g. [2], [3]. Due to that variety of possible casting defects, however, model-based approaches are unsuitable for the reconstruction of casting defects, or at least only suitable for a strongly limited class of defects. Other approaches are based on modelling the image as a binary Markov random field (MRF), e.g. [4]. The reconstruction consists of estimating the whole voxels of an object minimizing an error function depending on the measured projections and on the a priori choice of the image modelling assumptions. The main difficulties using such approaches are to achieve a reasonable fast and accurate minimization considering the very large number of unknowns.

Following a two-stage procedure for the three-dimensional reconstruction of casting defects from few radiographs is presented, which is based on the knowledge of the material parameters. This approach uses no special a priori assumption over the shape of the object under investigation. First, the complexity of the reconstruction problem is reduced by a limitation of the reconstruction to regions of interest around the defects. The defect areas are segmented and the extensions of the defects in beam directions are calculated from the measured data. In a second step the defects are reconstructed with an iterative tomographic procedure. Regularization of the reconstruction problem is achieved on the basis of the maximum entropy principle in connection with an iterative procedure for the binarisation of the reconstruction results. The basic idea of this approach is to use the maximum entropy solution as a distorted version of the true object and to impose the binary constrain in a second process, instead of imposing the binary constrain directly to the data. To improve the solution we iterate between the maximum entropy reconstruction and the segmentation process. Such an approach is computationally much less expensive than the direct estimation of the binary object from its projections.

2 Preprocessing

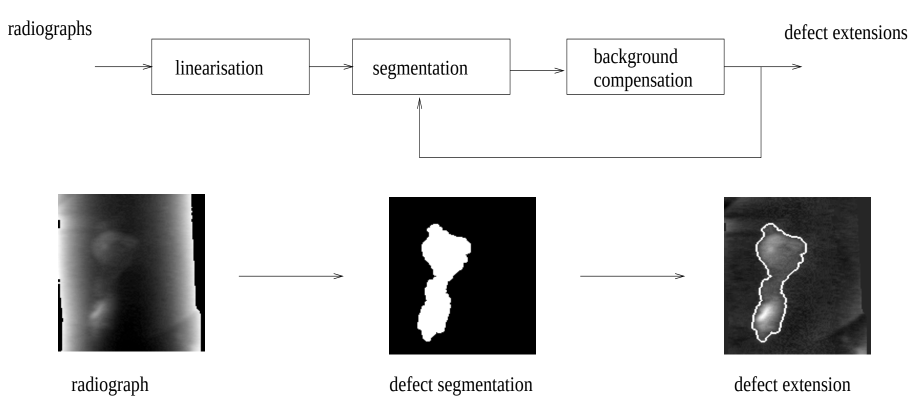

Figure 1: Preprocessing

The preprocessing is used to reduce the complexity of the reconstruction problem by restricting the reconstruction area to regions of interest around the defects. In figure 1 the principle of the preprocessing procedure is shown. The measured data are converted into material extensions in beam direction considering the known material parameters of the casting material (linearization). The defect areas are segmented (segmentation) and the material extension within the defect areas are extrapolated from the surrounding material expansions. Extrapolated and measured material extensions are compared within defect areas and the defect extensions are calculated (background compensation), compare [5]. The estimated defect extensions are used iteratively for an improvement of segmentation and background compensation.

3 Tomographic Reconstruction

The three-dimensional reconstruction of the defects is carried out on the basis of the preprocessing results. The object area is divided into discrete voxels and the projection of the individual voxels into the different radiographs is described with the help of projection matrix $\mathbf{A}$ (1).

$$\vec{p} = \mathbf{A} * \vec{\mu} \tag{1}$$

The data vector $\vec{p}$ contains the N pixel values of the different radiographs, in our case the defect extensions in beam direction calculated in the preprocessing step. The unknown vector $\vec{\mu}$ represents the M material values of the different voxels. Since the defect extensions in beam direction are used as input data, the material values of the voxels are limited to the value unity for a voxel, that belongs to a defect and the value zero for voxels outside the defect areas. The projection matrix $\mathbf{A}$ represents the projection characteristics of the accommodation system for the different radiographs. The matrix value a_{ij} corresponds to the influence of the i.th voxel on the j.th projection.

Using only few radiographs the set of equations (1) is underdetermined and at the same time inconsistent due to different error influences in the input data. Error influences are in particular errors in preprocessing, calibration errors both in the densiometric and in the geometrical calibration of the accommodation system as well as different noise sources. In general, the solution of such a problem can be defined as the minimizer of a compound criterion.

$$E = ||\mathbf{A} * \vec{\mu} - \vec{p}||^2 + \beta * E_R(\vec{\mu}) \tag{2}$$

$E_R(\vec{\mu})$ can be used to impose different regularization constrains on the solution, e.g. smoothness, binary constrains or assumptions about the shape of an object. β is the regularization factor controlling the influence of the regularization constrains. β is chosen according to the confidence in the measured data.

In our approach, we first calculate a smooth solution using the maximum entropy principle and impose the binary constrain in a second step, instead of imposing the binary constrain directly.

3.1 Maximum Entropy Principle

The maximum entropy principle is a criterion often used in different ranges of application for the determination of the solution of a underdetermined and inconsistent set of equations, e.g. [6], [7]. Entropy is a measure for the information content of the solution. In the sense of the maximum entropy principle that solution is optimal, which has the maximum entropy and thus the minimum information content of all possible solutions (3). The most interesting property of the maximum entropy solution is that it is maximally indefinite regarding to not measured projections, i.e. only such structures are reconstructed, which come out unique from the measured data.

The reconstructed values are not limited to zero or unity. The maximum entropy solution can be interpreted as a probability distribution, which indicates how probable a voxels belongs to the defect. The larger the value of a voxel is, the more probable it is that the voxel belongs to the defect.

$$\text{Maximize}$$

$$Entropy = -\sum_{i=0}^{M-1} \mu_i * \ln(\mu_i) \tag{3}$$

$$\begin{gathered}\text{according to}\\ \vec{p} = \mathbf{A} * \vec{\mu};\\ 0 <= \mu_i <= 1, \forall i \in \{0, M-1\}\end{gathered}$$

For the calculation of the solution different procedures are suggested in the literature. Own investigations [8] as well as the results of Subbarao [9] show that the MART algorithm supplies the best results for our application. The MART algorithm is based on an iterative adjustment of the reconstruction data to the measured data by multiplicative corrections. We found out that the basic MART algorithm can be improved using some additional steps according to the following procedure.

1. Start with a strictly positive vector $\vec{\mu}^0$
2. Calculate for all pixels p_j the actual projection:

$$\tilde{p}_j = \sum_k a_{kj} * \mu_k \tag{4}$$

3. Calculate for all voxels i and all pixels j the new voxel values:

$$\mu_i^{new} = \mu_i^{old} * \left(\frac{p_j}{\tilde{p}_j}\right)^{\lambda * a_{ij}} \tag{5}$$

4. Update all voxels according

$$\mu_i^{new} = \begin{cases} \mu_i^{old} & if \quad \mu_i^{old} \leq 1 \\ 1 & if \quad \mu_i^{old} > 1 \end{cases} \tag{6}$$

5. Update all voxel i every k.th iteration

$$\mu_i^{new} = median(\mu_i^{old}) \tag{7}$$

6. Repeat steps 4 to 7 until convergence

The first steps contain the basic MART algorithm. Depending on the noise properties of the data a small relaxation factor λ between 0.01 and 0.1 is selected in order to guarantee a stable convergence of the procedures. Step 6 imposes the material constrains on the reconstruction. Finally, the repeated median filtering of the intermediate results leads to a clear improvement of the convergence behavior [8]. The noise properties of the measured data used for the choice of an adequate relaxation factor are estimated during the preprocessing procedure.

3.2 Binary Constrains

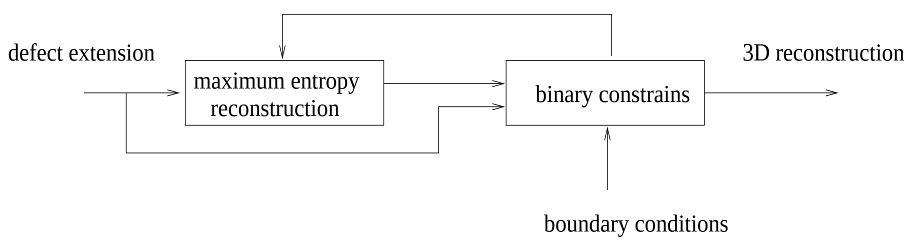

Figure 2: Overview over the reconstruction procedure

To impose the binary constrain the maximum entropy reconstruction is linked with an extended use of the a priori knowledge of the material parameters in a binarisation process. The flow of the developed reconstruction procedure is sketched in figure 2.

First the maximum entropy solution is calculated. In a binarisation process those voxels with the largest material values are identified as belonging to the defect. Sequently, a new solution of maximum entropy is calculated using the voxel identified as belongig to the defect as a boundary condition . This procedure is repeated iteratively up to a complete binary reconstruction. During the binarisation process a few natural geometrical boundary conditions are used, e.g. within a defect no material can be enclosed. If additional a priori assumptions over the shape of the defects are known, they are brought into the binarisation process as further boundary conditions. The basic idea of the procedure is to assign only few voxels to the defect in each iteration, in order to ensure a gradual adjustment of the solution to the binary constrain.

4 Results

To measure the performance of the proposed method experiments are carried out using simulated data records. A ball, a right parallelepiped and an object of complex geometry shown in figure 3 are used as test objects. The reconstructions are calculated from 5

	noise 0%	noise 5%	noise 10%
ball	0%	1%	3%
parallelepiped	0%	2%	3%
complex object	1.2%	5%	7%

Table 1: Reconstruction errors (simulated objects)

simulated radiographs distributed within a range of 90°. The position of the projections is varied to investigate the dependency of the results on the projection direction. The influence of measurement errors s investigated using Gaussian noise of different strength. The amount of falsely reconstructed voxel related to the volume of the object is used as an error measurement for the evaluation of the reconstruction .

In table 1 the results of the reconstruction using simulated radiographs are summarized. The given results are averaged over several attempts. Without noise influence an exact reconstruction is achieved for the ball and the right parallelepiped. The complex object is reconstructed with a small error of about 1.2%. Very small deviations of the reconstructed shape from the true shape occur depending on the selection of the projection directions.

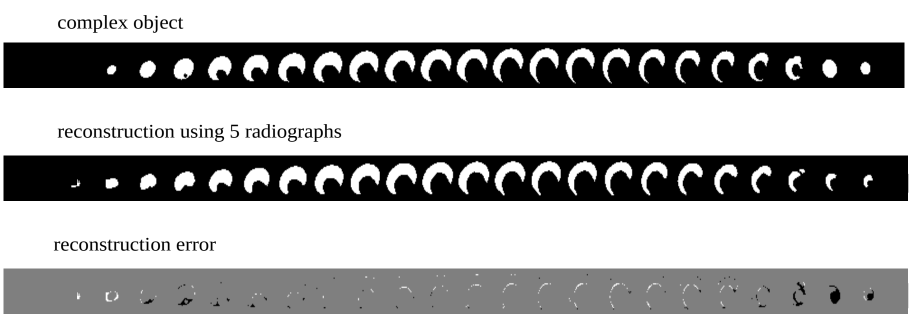

Figure 3: Reconstruction of a complex object

With increasing noise the number of falsely reconstructed voxel increases, the object shape however is shown in all cases only insignificantly worse than in the case of ideal data. In figure 3 the reconstruction result for the complex object using projections with 10% noise overlayed is shown layer by layer. Voxel, which are reconstructed incorrectly as belonging to the defect, are marked white in the presentation of the reconstruction error. Voxel, which are determined incorrectly as not belonging to the defect, are marked black.

In the following, investigations are carried out using defects of well-known geometry as well as real casting defects. The radiographs were taken up with a simple radioscopic system consisting of a microfocus X-ray tube, an image converter and a CDD camera. The calibration of the geometrical characteristics of the radioscopic system was achieved by using a special calibration procedure. Details of the calibration procedure are described in [5].

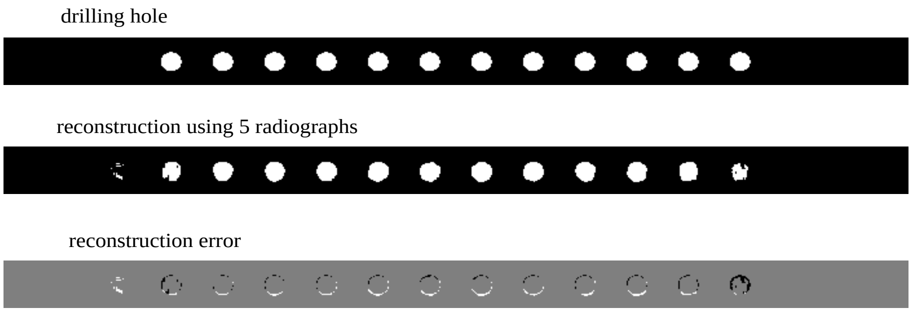

Figure 4: Reconstruction of a drilling hole

Defects of well-known geometry are simulated in a test sample by drilling holes of different size and depth. As an example, in figure 4 the reconstruction of a cylindrical drilling hole is presented. Using 5 radiographs in a 90° range, about 3% of the voxel are reconstructed wrong. The deviations can all be found at the surface of the hole. On the average the deviations of the reconstructed shape from the actual shape is less than one voxel. The maximum shape deviation amounts to two Voxel. These results have been confirmed using drilling holes of different shape and size.

The reconstruction of a real casting defect using 5 radiographs (90° range) is presented in figure 5. The maximum entropy solution and the binary reconstruction are shown.

These test results indicate that it is possible to reconstruct casting defects from only few radiographs using the material parameters of the casting samples as a priori knowledge.

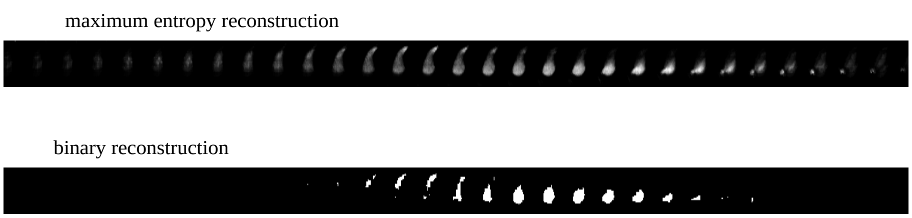

Figure 5: Reconstruction of a casting defect

5 Conclusions

An approach for the tomographic reconstruction of casting defects from few radiographs has been presented. The two-stage procedure is based on a systematic use of the a priori knowledge of the material parameters of the casting sample in connection with the limitation of the reconstruction to regions of interest around defect areas. For the regularization of the tomographic reconstruction problem the maximum entropy principle is used in connection with an iterative procedure for the binarisation of the reconstruction.

Investigations using simulated data as well as defects of well-known geometry show that the presented procedure enables a reconstruction from very few radiographs. A high reconstruction accuracy is achieved for simple defect geometries. The accuracy of the reconstruction of complex defects is examined at present in further investigations.

References

[1] Hanke,R.: Ein Verfahren zur Bestimmung von Lage und Umfang von Volumendefekten in Echtzeit am Beispiel der Gußteilepruefung. PhD thesis, University of Erlangen, Germany, 1996.

[2] Milanfar, P.; Karl, W.C.; Willesky, A.S.: Reconstructing Binary Polygonal Objects from Projections: A Statistical View. Graphical Models and Image Processing, 56, 5, 1994.

[3] Wallingford, R.M.; Basart, J.P.: Reconstruction of Ellipsoidal Flaw Models from X-Ray Radiographs. Review of Progress in Quantitative Nondestructive Evaluation, 10A, 1991.

[4] Retraint, F.; Peyrin, F.; Dintem, J.M.: Three-Dimensional Regularized Binary Image Reconstruction From Three Two-Dimensional Projections Using a Randomized ICM Algorithm. Int. J. Imaging Syst. Technol. 9, 1998, pp. 135-146.

[5] Lehr, C.; Feiste, K.L.; Stegemann, D.; Liedtke, C.-E.: Three dimensional defect analysis using stereoradioscopy based on camera modelling. 7th ECNDT, Copenhagen, May 1998.

[6] Reis, M.L.; Roberty, N.C.: Maximum entropy algorithm for image reconstruction from projections. Inverse Problems 8, 1992, pp. 623-644.

[7] Nailong Wu: The Maximum Entropy Method. Springer-Verlag Berlin, 1997.

[8] Eckert, G.: Untersuchung algebraisch iterativer Tomographieverfahren zur 3D-Rekonstruktion von Gußfehlern aus Rœntgenprojektionen. Studienarbeit, University of Hannover, Germany, 1998.

[9] Subbarao, P.M.V.; Munshi, P., Muralidhar, K.: Performance of iterative tomographic algorithms applied to non-destructive evaluation with limited data. NDT& E International, 30, 6,1997, pp. 359-370.